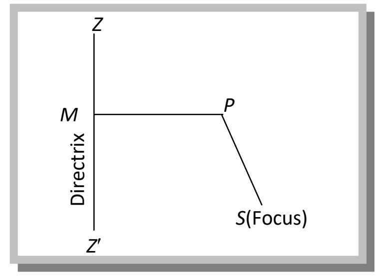
Definition

A hyperbola is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point in the same plane to its distance from a fixed line is always constant which is always greater than unity. Fixed point is called focus, fixed straight line is called directrix and the constant ratio is called eccentricity of the hyperbola. Eccentricity is denoted by *e* and *e*> 1.



A hyperbola is the particular case of the conic

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

When , $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$ *i.e.,* $\Delta \neq 0$ and $h^2 > ab$.

Let $S(h,k)$ is the focus, directrix is the line $ax + by + c = 0$ and the eccentricity is *e*. Let $P(x_1, y_1)$ be a point which moves such that $SP = e.PM$

$\Rightarrow \sqrt{(x_1 - h)^2 + (y_1 - k)^2} = e.\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$

$\Rightarrow (a^2 + b^2)[(x_1 - h)^2 + (y_1 - k)^2] = e^2(ax_1 + by_1 + c)^2$

Hence, locus of (x_1, y_1) is given by $(a^2 + b^2)[(x - h)^2 + (y - k)^2] = e^2(ax + by + c)^2$

Which is a second degree equation to represent a hyperbola (*e* > 1).