Intersection of a Circle and a Rectangular Hyperbola.

If a circle $x^2 + y^2 + 2gx + 2fy + k = 0$ cuts a rectangular hyperbola $xy = c^2$ in A, B, C and D and the parameters of these four points be t_1, t_2, t_3 and t_4 respectively; then

(1) (i) $\Sigma t_1 = -\frac{2g}{c}$ (ii) $\Sigma t_1 t_2 = \frac{k}{c^2}$

(iii) $\Sigma t_1 t_2 t_3 = \frac{-2f}{c}$ (iv) $t_1 t_2 t_3 t_4 = 1$ (v) $\Sigma \frac{1}{t_1} = -\frac{2f}{c}$

(2) Orthocentre of ΔABC is $H\left(-ct_4, \frac{-c}{t_4}\right)$ but D is $\left(ct_4, \frac{c}{t_4}\right)$

Hence *H* and *D* are the extremities of a diagonal of rectangular hyperbola.

(3) Centre of mean position of four points is $\left\{\frac{c}{4}\Sigma t_1, \frac{c}{4}\Sigma\left(\frac{1}{t_1}\right)\right\}$ *i.e.,* $\left(-\frac{g}{2}, -\frac{f}{2}\right)$

$\because$ Centers of the circles and rectangular hyperbola are (– *g*, – *f*) and (0, 0); midpoint of centers of circle and hyperbola is $\left(-\frac{g}{2}, -\frac{f}{2}\right)$. Hence the center of the mean position of the four points bisects the distance between the centers of the two curves (circle and rectangular hyperbola)

(4) If the circle passing through ABC meet the hyperbola in fourth points *D*, then center of circle is (–*g*, –*f*)

i.e., $\left\{\frac{c}{2}\left(t_1 + t_2 + t_3 + \frac{1}{t_1 t_2 t_3}\right); \frac{c}{2}\left(\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + t_1 t_2 t_3\right)\right\}$