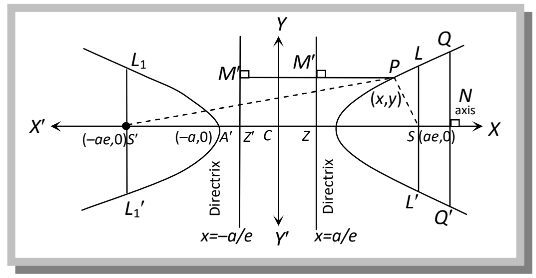
Standard equation of the Hyperbola.

Let *S* be the focus, *ZM* be the directrix and *e* be the eccentricity of the hyperbola, then by definition,



$$\Rightarrow \frac{SP}{PM} = e \Rightarrow (SP)^2 = e^2 (PM)^2$$

$$\Rightarrow (x - a.e)^2 + (y-0)^2 = e^2\left(x - \frac{a}{e}\right)^2$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1 \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$, where $b^2 = a^2(e^2-1)$

This is the standard equation of the hyperbola.

Some terms related to hyperbola: Let the equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(1) **Centre :**All chords passing through *C* are bisected at *C*. Here $C(0,0)$

(2) **Vertex:** The point *A* and *A*′ where the curve meets the line joining the foci *S* and *S*′ are called vertices of hyperbola. The co-ordinates of *A* and *A*′ are (*a*, 0) and (– *a*, 0) respectively.

(3) **Transverse and conjugate axes:** The straight line joining the vertices A and A′ is called transverse axis of the hyperbola. The straight line perpendicular to the transverse axis and passing through the centre is called conjugate axis.

Here, transverse axis = $AA' = 2a$

Conjugate axis = $BB' = 2b$

(4) **Eccentricity:** For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We have $b^2 = a^2(e^2-1)$, $e = \sqrt{1 + \left(\frac{2b}{2a}\right)^2} = \sqrt{1 + \left(\frac{\text{Conjugate axis}}{\text{Transverse axis}}\right)^2}$

(5) **Double ordinates:** If *Q* be a point on the hyperbola, *QN* perpendicular to the axis of the hyperbola and produced to meet the curve again at Q'. Then QQ' is called a double ordinate at *Q*.

If abscissa of Q is h, then co-ordinates of Q and Q' are $\left(h, \frac{b}{a}\sqrt{h^2 - a^2}\right)$ and $\left(h, -\frac{b}{a}\sqrt{h^2 - a^2}\right)$ respectively.

(6) **Latus-rectum:** The chord of the hyperbola which passes through the focus and is perpendicular to its transverse axis is called latus-rectum.

Length of latus-rectum $LL' = L_1L_1' = \frac{2b^2}{a} = 2a(e^2 - 1)$ and end points of latus-rectum $L\left(ae, \frac{b^2}{a}\right)$; $L'\left(ae, \frac{-b^2}{a}\right)$; $L_1\left(-ae, \frac{b^2}{a}\right)$; $L_1'\left(-ae, -\frac{b^2}{a}\right)$ respectively.

(7) **Foci and directrices:** The points $S(ae, 0)$ and $S'(-ae, 0)$ are the foci of the hyperbola and ZM and $Z'M'$ are two directrices of the hyperbola and their equations are $x = \frac{a}{e}$ and $x = -\frac{a}{e}$ respectively.

Distance between foci $SS' = 2ae$ and distance between directrices $ZZ' = 2a/e$.

(8) **Focal chord:** A chord of the hyperbola passing through its focus is called a focal chord.

(9) **Focal distance:** The difference of any point on the hyperbola from the focus is called the focal distance of the point.

From the figure, $SP = ePM = e\left(x_1 - \frac{a}{e}\right) = ex_1 - a$, $S'P = ePM' = e\left(x_1 + \frac{a}{e}\right) = ex_1 + a$

The difference of the focal distance of a point on the hyperbola is constant and is equal to the length of transverse axis.

$|S'P - SP| = 2a = AA' =$ Transverse axis