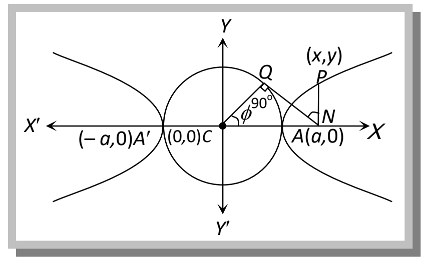
Auxiliary circle of Hyperbola.



Let $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be the hyperbola with center C and transverse axis $A'A$.

Therefore circle drawn with center C and segment $A'A$ as a diameter is called auxiliary circle of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore$ Equation of the auxiliary circle is $x^2+y^2=a^2$

Let $\angle QCN=\phi$

Here P and Q are the corresponding points on the hyperbola and the auxiliary circle $(0\le\phi<2\pi)$

(1) **Parametric equations of hyperbola:** The equations $x=a\sec\phi$ and $y=b\tan\phi$ are known as the parametric equations of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. This $(a\sec\phi, b\tan\phi)$ lies on the hyperbola for all values of ϕ.

Position of points Q on auxiliary circle and the corresponding point P which describes the hyperbola and $0\le\phi<2\pi$		
ϕ varies from	$Q(a\cos\varphi, a\sin\varphi)$	$P(a\sec\varphi, b\tan\varphi)$
0 to $\frac{\pi}{2}$	I	I
$\frac{\pi}{2}$ to π	II	III
π to $\frac{3\pi}{2}$	III	II

$\frac{3\pi}{2}$ to 2π	IV	IV

Note: The equations $x = a\cosh\theta$ and $y = b\sin\text{h}\,\theta$ are also known as the parametric equations of the hyperbola and the co-ordinates of any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are expressible as $(a\cos\text{h}\,\theta,\ b\sin\text{h}\,\theta)$, where $\cos\text{h}\,\theta = \frac{e^{\theta} + e^{-\theta}}{2}$ and $\sin\text{h}\,\theta = \frac{e^{\theta} - e^{-\theta}}{2}$.