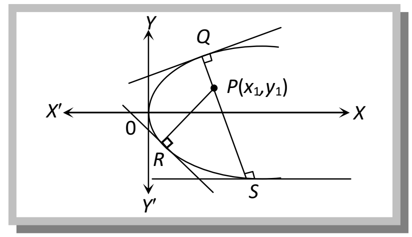
Co-normal Points.

The points on the curve at which the normals pass through a common point are called co-normal points.



Q, R, S are co-normal points. The co- normal points are also called the feet of the normals.

If the normal passes through point $P(x_1, y_1)$ which is not on parabola, then

$y_1 = mx_1 - 2am - am^3 \Rightarrow am^3 + (2a - x_1)m + y_1 = 0$ ......(i)

Which gives three values of m**.** Let three values of m are m_1, m_2 and m_3, which are the slopes of the normals at Q, R and S respectively, then the coordinates of Q, R and S are $(am_1^2, -2am_1), (am_2^2, -2am_2)$ and $(am_{3,}^2 - 2am_3)$ respectively. These three points are called the feet of the normals.

Now $m_1 + m_2 + m_3 = 0$, $m_1m_2 + m_2m_3 + m_3m_1 = \frac{(2a - x_1)}{a}$ and $m_1m_2m_3 = \frac{-y_1}{a}$

In general, three normals can be drawn from a point to a parabola.

(1) The algebraic sum of the slopes of three concurrent normals is zero.

(2) The sum of the ordinates of the co-normal points is zero.

(3) The centroid of the triangle formed by the co-normal points lies on the axis of the parabola.

(4) The centroid of a triangle formed by joining the foots of the normal of the parabola lies on its axis and is given by $\left(\frac{am_1^2 + am_2^2 + am_3^2}{3}, \frac{2am_1 + 2am_2 + 2am_3}{3}\right) = \left(\frac{am_1^2 + am_2^2 + am_3^2}{3}, 0\right)$

(5) If three normals drawn to any parabola $y^2 = 4ax$ from a given point (h, k) be real, then $h > 2a$ for $a = 1$, normals drawn to the parabola $y^2 = 4x$ from any point (h, k) are real, if $h > 2$.

(6) Out of these three at least one is real, as imaginary normals will always occur in pairs.