Equation of the Chord joining any two points on the Parabola.

Let $P(at_1^2, 2at_1), Q(at_2^2, 2at_2)$ be any two points on the parabola $y^2 = 4ax$. Then, the equation of the chord joining these points is, $y - 2at_1 = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1^2)$ or $y - 2at_1 = \frac{2}{t_1 + t_2}\left(x - at_1^2\right)$ or $y(t_1 + t_2) = 2x + 2at_1t_2$

(1) **Condition for the chord joining points having parameters t_1 and t_2 to be a focal chord:** If the chord joining points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola passes through its focus, then $(a, 0)$ satisfies the equation $y(t_1 + t_2) = 2x + 2at_1t_2 \Rightarrow 0 = 2a + 2at_1t_2 \Rightarrow t_1t_2 = -1$ or $t_2 = -\frac{1}{t_1}$

(2) **Length of the focal chord:** The length of a focal chord having parameters t_1 and t_2 for its end points is $a(t_2 - t_1)^2$.

Note: If one extremity of a focal chord is $(at_1^2, 2at_1)$, then the other extremity $(at_2^2, 2at_2)$ becomes $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$ by virtue of relation $t_1t_2 = -1$.

If one end of the focal chord of parabola is $(at^2, 2at)$, then other end will be $\left(\frac{a}{t^2}, -2at\right)$ and length of chord $= a\left(t + \frac{1}{t}\right)^2$.

The length of the chord joining two point $'t_1'$ and $'t_2'$ on the parabola $y^2 = 4ax$ is $a(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4}$

The length of intercept made by line $y = mx + c$ between the parabola $y^2 = 4ax$ is $\frac{4}{m^2}\sqrt{a(1 + m^2)(a - mc)}$.

Important Tips

☞ The focal chord of parabola $y^2 = 4ax$ making an angle α with the x-axis is of length $4a\,\mathrm{cosec}^2\alpha$.

☞ The length of a focal chord of a parabola varies inversely as the square of its distance from the vertex.

- If l_1 and l_2 are the length of segments of a focal chord of a parabola, then its latus-rectum is $\frac{4l_1l_2}{l_1+l_2}$
- The semi latus rectum of the parabola $y^2 = 4ax$ is the harmonic mean between the segments of any focal chord of the parabola.