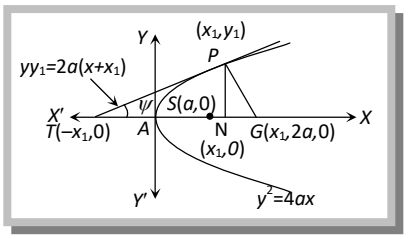
Length of Tangent, Sub tangent, Normal and Subnormal.

Let the parabola $y^2 = 4ax$. Let the tangent and normal at $P(x_1, y_1)$ meet the axis of parabola at T and Grespectively, and tangent at $P(x_1, y_1)$ makes angle ψ with the positive direction of x-axis.

$A(0,0)$ is the vertex of the parabola and $PN = y$. Then,



(1) Length of tangent $= PT = PN \operatorname{cosec} \psi = y_1 \operatorname{cosec} \psi$

(2) Length of normal $= PG = PN \operatorname{cosec}(90^o - \psi) = y_1 \sec \psi$

(3) Length of subtangent $= TN = PN \cot \psi = y_1 \cot \psi$

(4) Length of subnormal $= NG = PN \cot(90^o - \psi) = y_1 \tan \psi$

where , $\tan \psi = \dfrac{2a}{y_1} = m$, [slope of tangent at P(x, y)]

Length of tangent, subtangent, normal and subnormal to $y^2 = 4ax$ at $(at^2, 2at)$

(1) Length of tangent at $(at^2, 2at) = 2at \operatorname{cosec} \psi = 2at\sqrt{(1 + \cot^2 \psi)} = 2at\sqrt{1 + t^2}$

(2) Length of normal at $(at^2, 2at) = 2at \sec \psi = 2at\sqrt{(1 + \tan^2 \psi)} = 2a\sqrt{t^2 + t^2 \tan^2 \psi} = 2a\sqrt{(t^2 + 1)}$

(3) Length of subtangent at $(at^2, 2at) = 2at \cot \psi = 2at^2$

(4) Length of subnormal at $(at^2, 2at) = 2at \tan \psi = 2a$