

Characteristics of Pole and Polar.

(1) **Conjugate points:** If the polar of $P(x_1, y_1)$ passes through $Q(x_2, y_2)$, then the polar of $Q(x_2, y_2)$ goes through $P(x_1, y_1)$ and such points are said to be conjugate points.

Two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are conjugate points with respect to the parabola $y^2 = 4ax$, if $y_1 y_2 = 2a(x_1 + x_2)$.

(2) **Conjugate lines:** If the pole of a line $ax + by + c = 0$ lies on the line $a_1 x + b_1 y + c_1 = 0$, then the pole of the second line will lie on the first and such lines are said to be conjugate lines.

Two lines $l_1 x + m_1 y + n_1 = 0$ and $l_2 x + m_2 y + n_2 = 0$ are conjugate lines with respect to parabola $y^2 = 4ax$, if $(l_1 n_2 + l_2 n_1) = 2am_1 m_2$

Note: The chord of contact and polar of any point on the directrix always passes through focus.

The pole of a focal chord lies on directrix and locus of poles of focal chord is the directrix.

The polars of all points on directrix always pass through a fixed point and this fixed point is focus.