

Standard equation of the Parabola.

Let S be the focus ZZ' be the directrix of the parabola and (x,y) be any point on parabola.

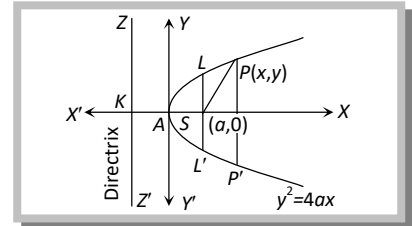
Let $AS = AK = a (> 0)$ then coordinate of S is $(a, 0)$ and the equation of KZ is $x = -a$ or $x + a = 0$

Now $SP = PM \Rightarrow (SP)^2 = (PM)^2$

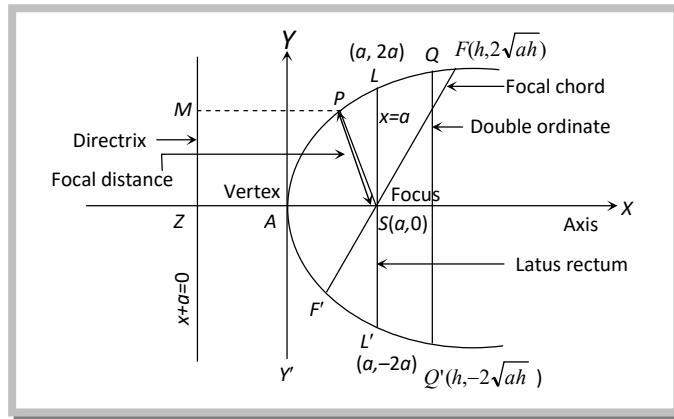
$$\Rightarrow (x - a)^2 + (y - 0)^2 = (a + x)^2$$

$$\therefore \boxed{y^2 = 4ax}$$

Which is the equation of the parabola in its standard form.



Some terms related to parabola



For the parabola $y^2 = 4ax$,

(1) **Axis:** A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola.

For the parabola $y^2 = 4ax$, x -axis is the axis. Here all powers of y are even in $y^2 = 4ax$. Hence parabola $y^2 = 4ax$ is symmetrical about x -axis.

(2) **Vertex:** The point of intersection of a parabola and its axis is called the vertex of the parabola. The vertex is the middle point of the focus and the point of intersection of axis and the directrix. For the parabola $y^2 = 4ax$, $A(0,0)$ i.e., the origin is the vertex.

(3) **Double-ordinate:** The chord which is perpendicular to the axis of parabola or parallel to directrix is called double ordinate of the parabola.

Let QQ' be the double-ordinate. If abscissa of Q is h then ordinate of Q , $y^2 = 4ah$ or $y = 2\sqrt{ah}$ (for I^{st} Quadrant) and ordinate of Q' is $y = -2\sqrt{ah}$ (for IV^{th} Quadrant). Hence coordinates of Q and Q' are $(h, 2\sqrt{ah})$ and $(h, -2\sqrt{ah})$ respectively.

(4) **Latus-rectum:** If the double-ordinate passes through the focus of the parabola, then it is called latus-rectum of the parabola.

Coordinates of the extremities of the latus rectum are $L(a, 2a)$ and $L'(a, -2a)$ respectively.

Since $LS = L'S = 2a \therefore$ Length of latusrectum $LL' = 2(LS) = 2(L'S) = 4a$.

(5) **Focal Chord:** A chord of a parabola which is passing through the focus is called a focal chord of the parabola. Here PP' and LL' are the focal chords.

(6) **Focal distance (Focal length):** The focal distance of any point P on the parabola is its distance from the focus *S*, i.e., SP .

Here, Focal distance $SP = PM = x + a$

Note : \square If length of any double ordinate of parabola $y^2 = 4ax$ is $2l$, then coordinates of end points of this double ordinate are $\left(\frac{l^2}{4a}, l\right)$ and $\left(\frac{l^2}{4a}, -l\right)$.

Important Tips

☞ The area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$, where y_1, y_2, y_3 are the ordinate of the vertices

☞ The length of the side of an equilateral triangle inscribed in the parabola $y^2 = 4ax$ is $8a\sqrt{3}$ (one angular point is at the vertex).