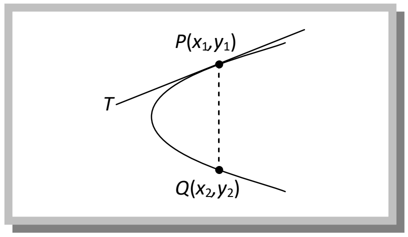
Equation of Tangent in Different forms.

(1) **Point Form:** The equation of the tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) is $yy_1 = 2a(x + x_1)$



Equation of tangent of all other standard parabolas at (x₁, y₁)	
Equation of parabolas	**Tangent at (x_1, y_1)**
$y^2 = -4ax$	$yy_1 = -2a(x + x_1)$
$x^2 = 4ay$	$xx_1 = 2a(y + y_1)$
$x^2 = -4ay$	$xx_1 = -2a(y + y_1)$

Note: The equation of tangent at (x_1, y_1) to a curve can also be obtained by replacing x^2 by xx_1, y^2 by yy_1, x by $\frac{x + x_1}{2}$, y by $\frac{y + y_1}{2}$ and xy by $\frac{xy_1 + x_1 y}{2}$ provided the equation of curve is a polynomial of second degree in x and y.

(2) **Parametric form:** The equation of the tangent to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $ty = x + at^2$

Equations of tangent of all other standard parabolas at 't'		
Equations of parabolas	**Parametric co-ordinates**	**Tangent at 't'**

	't'	
$y^2 = -4ax$	$(-at^2, 2at)$	$ty = -x + at^2$
$x^2 = 4ay$	$(2at, at^2)$	$tx = y + at^2$
$x^2 = -4ay$	$(2at, -at^2)$	$tx = -y + at^2$

(3) **Slope Form:** The equation of a tangent of slope m to the parabola $y^2 = 4ax$ $at\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ is

$$y = mx + \frac{a}{m}$$

Equation of parabolas	**Point of contact in terms of slope (m)**	**Equation of tangent in terms of slope (m)**	**Condition of Tangency**
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$	$c = \frac{a}{m}$
$y^2 = -4ax$	$\left(-\frac{a}{m^2}, -\frac{2a}{m}\right)$	$y = mx - \frac{a}{m}$	$c = -\frac{a}{m}$
$x^2 = 4ay$	$(2am, am^2)$	$y = mx - am^2$	$c = -am^2$
$x^2 = -4ay$	$(-2am, -am^2)$	$y = mx + am^2$	$c = am^2$

Important Tips

- If the straight line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ then $ln = am^2$.
- If the line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $y^2 = 4ax$, then $P\cos\alpha + a\sin^2\alpha = 0$ and point of contact is $(a\tan^2\alpha, -2a\tan\alpha)$

☞ If the line $\frac{x}{l}+\frac{y}{m}=1$ touches the parabola $y^2 = 4a(x+b)$, then $m^2(l+b)+al^2 = 0$