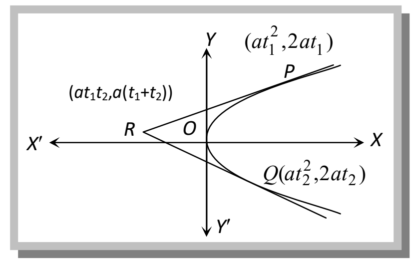
Point of intersection of Tangents at any two points on the Parabola.

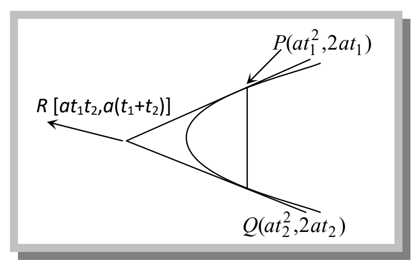
The point of intersection of tangents at two points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is $(at_1t_2, a(t_1 + t_2))$.



The locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ which meet at an angle α is $(x+a)^2 \tan^2 \alpha = y^2 - 4ax$.

Director circle: The locus of the point of intersection of perpendicular tangents to a conic is known as its director circle. The director circle of a parabola is its directrix.

Note: Clearly, x-coordinates of the point of intersection of tangents at P and Q on the parabola is the G.M of the x-coordinate of P and Q and y-coordinate is the A.M. of y-coordinate of P and Q.



The equation of the common tangents to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is

$$a^{\frac{1}{3}}x + b^{\frac{1}{3}}y + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$$

The tangents to the parabola $y^2 = 4ax$ at $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ intersect at R.

Then the area of triangle PQR is $\frac{1}{2}a^2(t_1 - t_2)^3$