1. Types of Sets.

(1) **Null setorEmpty set:** The set which contains no element at all is called the null set. This set is sometimes also called the 'empty set' or the 'void set'. It is denoted by the symbol ϕ or {}.

A set which has at least one element is called a non-empty set.

Let $A = \{x : x^2 + 1 = 0 \text{ and } x$ is real)

Since there is no real number which satisfies the equation $x^2 + 1 = 0$, therefore the set A is empty set.

Note: If A andB are any two empty sets, then $x \in A$ iff $x \in B$ is satisfied because there is no element x in either A orB to which the condition may be applied. Thus A = B. Hence, there is only one empty set and we denote it by ϕ. Therefore, article 'the' is used before empty set.

(2) **Singleton set:** A set consisting of a single element is called a singleton set. The set {5} is a singleton set.

(3) **Finite set:** A set is called a finite set if it is either void set or its elements can be listed (counted, labelled) by natural number 1, 2, 3, … and the process of listing terminates at a certain natural number n (say).

Cardinal number of a finite set: The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by n(A) or O(A).

(4) **Infinite set:** A set whose elements cannot be listed by the natural numbers 1, 2, 3, …., n, for any natural number n is called an infinite set.

(5) **Equivalent set:** Two finite sets A and B are equivalent if their cardinal numbers are same i.e. n (A) = n(B).

Example: $A = \{1, 3, 5, 7\}$; $B = \{10,\ 12, 14, 16\}$ are equivalent sets $[\because\ O(A) = O(B) = 4]$

(6) **Equal set:** Two sets A and B are said to be equal iff every element of A is an element of B and also every element of B is an element of A. We write "A = B" if the sets A and B are equal and "A $\neq$ B" if the sets A and B are not equal. Symbolically, A = B if x $\in$ A $\Leftrightarrow$ x $\in$ B.

The statement given in the definition of the equality of two sets is also known as the axiom of extension.

Example: If $A = \{2, 3, 5, 6\}$ and $B = \{6, 5, 3, 2\}$. Then $A = B,$ because each element of A is an element of B and vice-versa.

Note: Equal sets are always equivalent but equivalent sets may need not be equal set.

(7) **Universal set:**A set that contains all sets in a given context is called the universal set.

or

A set containing of all possible elements which occur in the discussion is called a universal set and is denoted by U.

Thus in any particular discussion, no element can exist out of universal set. It should be noted that universal set is not unique. It may differ in problem to problem.

(8) **Power set**: If S is any set, then the family of all the subsets of S is called the power set of S. The power set of S is denoted by P(S). Symbolically, P(S) = {T: T $\subseteq$ S}. Obviously ϕ and S are both elements of P(S).

Example**:** Let S = {a, b, c}, then P(S) = {ϕ , {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}.

Note: ❑ If $A = \phi,$ then $P(A)$ has one element ϕ , $\therefore n[P(A)] = 1$

❑ Power set of a given set is always non-empty.

❑ If A has n elements, then P (A) has 2^n elements.

❑ $P(\phi) = \{\phi\}$

$P(P(\phi)) = \{\phi, \{\phi\}\} \Rightarrow P[P(P(\phi))] = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Hence $n\{P[P(P(\phi))]\} = 4$.

(9) **Subsets (Set inclusion):**Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B.

If A is subset of B, we write A $\subseteq$ B, which is read as "A is a subset of B" or "A is contained in B".

Thus, A $\subseteq$ B $\Rightarrow$ a $\in$ A $\Rightarrow$ a $\in$ B.

Note: ❑ Every set is a subset of itself.

❑ The empty set is a subset of every set.

❑ The total number of subset of a finite set containing n elements is 2^n.

Proper and improper subsets: If A is a subset of B and $A \neq B,$ then A is a proper subset of B. We write this as $A \subset B$.

The null set ϕ is subset of every set and every set is subset of itself, i.e., $\phi \subset A$ and $A \subseteq A$ for every set A. They are called improper subsets of A. Thus every non-empty set has two improper subsets. It should be noted that ϕ has only one subset ϕ which is improper. Thus A has two improper subsets iff it is non-empty.

All other subsets of A are called its proper subsets. Thus, if $A \subset B,$ $A \neq B$, $A \neq \phi$, then A is said to be proper subset of B.

Example: Let $A = \{1, 2\}$. Then A has $\phi; \{1\}, \{2\}, \{1, 2\}$ as its subsets out of which ϕ and {1, 2} are improper and {1} and {2} are proper subsets.