

1. Laws of Algebra of Sets.

(1) **Idempotent laws:** For any set A, we have

$$(i) A \cup A = A \qquad (ii) A \cap A = A$$

(2) **Identity laws:** For any set A, we have

$$(i) A \cup \phi = A \qquad (ii) A \cap U = A$$

i.e. ϕ and U are identity elements for union and intersection respectively.

(3) **Commutative laws:** For any two sets A and B, we have

$$(i) A \cup B = B \cup A \qquad (ii) A \cap B = B \cap A \qquad (iii) A \Delta B = B \Delta A$$

i.e. union, intersection and symmetric difference of two sets are commutative.

$$(iv) A - B \neq B - A \qquad (v) A \times B \neq B \times A$$

i.e., difference and Cartesian product of two sets are not commutative

(4) **Associative laws:** If A, B and C are any three sets, then

$$(i) (A \cup B) \cup C = A \cup (B \cup C) \quad (ii) A \cap (B \cap C) = (A \cap B) \cap C \quad (iii) (A \Delta B) \Delta C = A \Delta (B \Delta C)$$

i.e., union, intersection and symmetric difference of two sets are associative.

$$(iv) (A - B) - C \neq A - (B - C) \quad (v) (A \times B) \times C \neq A \times (B \times C)$$

i.e., difference and Cartesian product of two sets are not associative.

(5) **Distributive law:** If A, B and C are any three sets, then

$$(i) A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad (ii) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

i.e. union and intersection are distributive over intersection and union respectively.

$$(iii) A \times (B \cap C) = (A \times B) \cap (A \times C) \quad (iv) A \times (B \cup C) = (A \times B) \cup (A \times C) \quad (v) \\ A \times (B - C) = (A \times B) - (A \times C)$$

(6) **De-Morgan's law:** If A and B are any two sets, then

$$(i) (A \cup B)' = A' \cap B' \qquad (ii) (A \cap B)' = A' \cup B'$$

$$(iii) A - (B \cup C) = (A - B) \cap (A - C) \quad (iv) A - (B \cap C) = (A - B) \cup (A - C)$$

Note: **Theorem 1:** If A and B are any two sets, then

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $A - B = A \Leftrightarrow A \cap B = \phi$

(iv) $(A - B) \cup B = A \cup B$

(v) $(A - B) \cap B = \phi$

(vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

(viii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Theorem 2: If A, B and C are any three sets, then

(i) $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(iv) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$