1. Cartesian Product of Sets.

Cartesian product of sets: Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the Cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = [(a, b): a \in A \text{ and } b \in B]$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$.

Example: Let A = {a, b, c} and B = {p, q}.

Then A × B = {(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)}

Also B × A = {(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)}

Important theorems on Cartesian product of sets:

Theorem 1: For any three sets A, B, C

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Theorem 2: For any three sets A, B, C

$$A \times (B - C) = (A \times B) - (A \times C)$$

Theorem 3: If A and B are any two non-empty sets, then

$$A \times B = B \times A \Leftrightarrow A = B$$

Theorem 4: If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

Theorem 5: If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

Theorem 6: If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

Theorem 7: For any sets A, B, C, D

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

Theorem 8: For any three sets A, B, C

$A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

(ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

Theorem 9: Let A and B two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.