

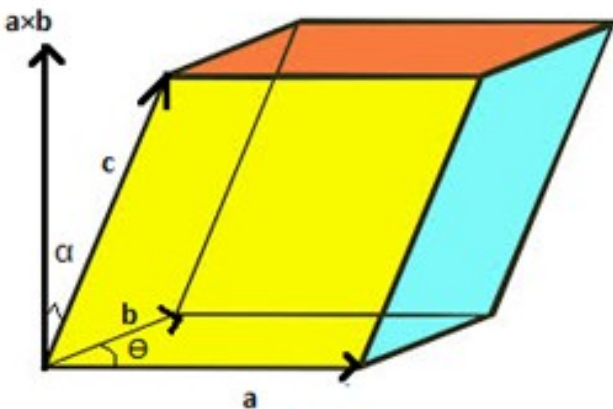
SCALAR TRIPLE PRODUCT

By the name itself, it is evident that scalar triple product of vectors means the product of three vectors. It means taking the **dot product** of one of the vectors with the cross product of the remaining two. It is denoted as

$$[a \ b \ c] = (a \times b) \cdot c$$

The following conclusions can be drawn, by looking into the above formula:

- i) The resultant is always a scalar quantity.
- ii) Cross product of the vectors is calculated first followed by the dot product which gives the scalar triple product.
- iii) Talking about the physical significance of scalar triple product formula it represents the volume of the parallelepiped whose three co-terminous edges represent the three vectors a, b and c . The following figure will make this point more clear.



According to this figure, the three vectors are represented by the coterminous edges as shown. The cross product of vectors a and b gives the area of the base and also the direction of the cross product of vectors is perpendicular to both the vectors. As volume is the product of area and height, the height in this case is given by the component of vector c along the direction of cross product of a and b . The component is given by $c \cos \alpha$.

Thus, we can conclude that for a parallel piped, if the coterminous edges are denoted by three vectors and a, b and c then,

$$\text{Volume of parallelepiped} = (a \times b) c \cos \alpha = (a \times b) \cdot c$$

Where α is the angle between $(a \times b)$ and c

We are familiar with the expansion of cross product of vectors. Keeping that in mind, if it is given that $a = x_1i^{\wedge} + a_2j^{\wedge} + a_3k^{\wedge}$, $b = b_1i^{\wedge} + b_2j^{\wedge} + b_3k^{\wedge}$, and $c = c_1i^{\wedge} + c_2j^{\wedge} + c_3k^{\wedge}$ then, we can express the above equation as,

$$(a \times b) \cdot c = \begin{vmatrix} i^{\wedge} & j^{\wedge} & k^{\wedge} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \cdot (c_1i^{\wedge} + c_2j^{\wedge} + c_3k^{\wedge})$$

This indicates the dot product of two vectors. Using properties of determinants, we can expand the above equation as,

$$= \mathbf{(a \times b) \cdot c}$$

$$= \mathbf{|||i \cdot (c_1i + c_2j + c_3k) a_1 b_1 j \cdot (c_1i + c_2j + c_3k) a_2 b_2 k \cdot (c_1i + c_2j + c_3k) a_3 b_3|||}$$

According to the dot product of vector properties,

$$i \cdot i = j \cdot j = k \cdot k = 1 \quad (\text{As } \cos 0 = 1)$$

$$\Rightarrow i \cdot (c_1i + c_2j + c_3k) = c_1$$

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$$\Rightarrow \mathbf{(a \times b) \cdot c = |||c_1 a_1 b_1 c_2 a_2 b_2 c_3 a_3 b_3|||}$$

$$\mathbf{[a \ b \ c] = |||c_1 a_1 b_1 c_2 a_2 b_2 c_3 a_3 b_3|||}$$

Properties of **Scalar Triple Product**:

i) If the vectors are cyclically permuted, then

$$\mathbf{(a \times b) \cdot c = a \cdot (b \times c)}$$

ii) The product is cyclic in nature, i.e.,

$$\mathbf{[a \ b \ c] = [b \ c \ a] = [c \ a \ b] = -[b \ a \ c] = -[c \ b \ a] = -[a \ c \ b]}$$

Example: Three vectors are given by, $\mathbf{a = i - j + k}$, $\mathbf{b = 2i + j + k}$, and $\mathbf{c = i - j - 2k}$.
By using the scalar triple product of vectors, verify that $\mathbf{[a \ b \ c] = [b \ c \ a] = -[a \ c \ b]}$

Solution: First of all let us find $\mathbf{[a \ b \ c]}$.

$$\mathbf{[a \ b \ c] = (a \times b) \cdot c}$$

We know $\mathbf{[a \ b \ c] = |||c_1 a_1 b_1 c_2 a_2 b_2 c_3 a_3 b_3|||}$

$$\Rightarrow \mathbf{[a \ b \ c] = |||1 \ 1 \ 2 \ 1 \ -1 \ 1 \ -2 \ 1 \ 1|||}$$

$$\Rightarrow \mathbf{[a \ b \ c] = -7}$$

Now let us evaluate $\mathbf{[b \ c \ a]}$ and $\mathbf{[a \ c \ b]}$ similarly,

$$\Rightarrow \mathbf{[b \ c \ a] = |||a_1 b_1 c_1 a_2 b_2 c_2 a_3 b_3 c_3||| = -7}$$

$$\Rightarrow \mathbf{[a \ c \ b] = |||b_1 a_1 c_1 b_2 a_2 c_2 b_3 a_3 c_3||| = 7}$$

Hence it can be seen that $\mathbf{[a \ b \ c] = [b \ c \ a] = -[a \ c \ b]}$

Try to recall about the properties of determinants using that this problem becomes easier to solve.

iii) If the triple product of vectors is zero, then it can be inferred that the vectors are coplanar in nature.

The triple product indicates the volume of a parallelepiped. If it is zero, then such a case could only arise when any one of the three vectors is of zero magnitude. The direction of the cross product of \mathbf{a} and \mathbf{b} is perpendicular to the plane which contains \mathbf{a} and \mathbf{b} . The dot product of the resultant with \mathbf{c} will only be zero if the vector \mathbf{c} also lies in the same plane. This is because the angle between the resultant and \mathbf{C} will be 90° and $\cos 90^\circ = 0$.

Thus, by the use of scalar triple product we can easily find out the volume of a given parallelepiped. There are a lot of real life applications of vectors which are very interesting to learn. To learn more on vectors, download Byju's learning app