a) Scalar Product of Four Vectors: The products already considered are usually sufficient for practical applications. But we occasionally meet with products of four vectors of the following types. Consider the scalar product of $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$. This is a number easily expressible in terms of the scalar products of the individual vectors. For, in virtue of the fact that in a scalar triple product the dot and cross may be interchanged, we may write

$$(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = \vec{a} \cdot \vec{b} \times (\vec{c} \times \vec{d}) = \vec{a} \cdot ((\vec{b} \cdot \vec{d})\vec{c} - (\vec{b} \cdot \vec{c})\vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

Writing this result in the form of a determinant,

we have $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix}$