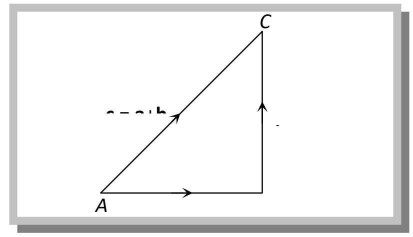
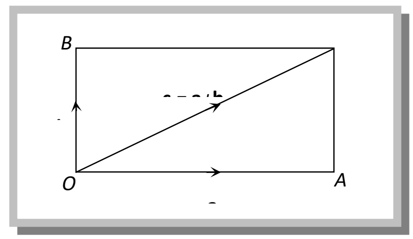
Properties of Vectors.

(1) **Addition of vectors**



(i) **Triangle law of addition:** If two vectors are represented by two consecutive sides of a triangle then their sum is represented by the third side of the triangle, but in opposite direction. This is known as the triangle law ofaddition of vectors. Thus, if $\overrightarrow{AB} = \mathbf{a}, \overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$ then $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ *i.e.,* $\mathbf{a} + \mathbf{b} = \mathbf{c}$.

(ii) **Parallelogram law of addition:** If two vectors are represented by two adjacent sides of a parallelogram, then their sum is represented by the diagonal of the parallelogram whose initial point is the same as the initial point of the given vectors. This is known as parallelogram law of addition of vectors.



Thus, if $\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$

Then $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$ *i.e.,* $\mathbf{a} + \mathbf{b} = \mathbf{c}$, where *OC* is a diagonal of the parallelogram *OABC*.

(iii) **Addition in component form :** If the vectors are defined in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, *i.e.,* if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then their sum is defined as

$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{i} + (a_2 + b_2)\mathbf{j} + (a_3 + b_3)\mathbf{k}$.

Properties of vector addition: Vector addition has the following properties.

(a) **Binary operation:** The sum of two vectors is always a vector.

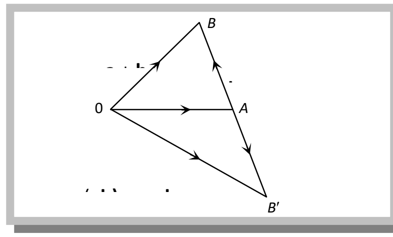
(b) **Commutativity:** For any two vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

(c) **Associativity:** For any three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$

(d) **Identity:** Zero vector is the identity for addition. For any vector $\mathbf{a}$, $\mathbf{0} + \mathbf{a} = \mathbf{a} = \mathbf{a} + \mathbf{0}$

(e) **Additive inverse:** For every vector $\mathbf{a}$ its negative vector $-\mathbf{a}$ exists such that $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$ *i.e.,* $(-\mathbf{a})$ is the additive inverse of the vector **a**.

(2) **Subtraction of vectors:** If $\mathbf{a}$ and $\mathbf{b}$ are two vectors, then their subtraction $\mathbf{a}-\mathbf{b}$ is defined as $\mathbf{a}-\mathbf{b}=\mathbf{a}+(-\mathbf{b})$ where $-\mathbf{b}$ is the negative of $\mathbf{b}$ having magnitude equal to that of $\mathbf{b}$ and direction opposite to $\mathbf{b}$.



If $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$

Then $\mathbf{a}-\mathbf{b}=(a_1-b_1)\mathbf{i}+(a_2-b_2)\mathbf{j}+(a_3-b_3)\mathbf{k}$.

Properties of vector subtraction

(i) $\mathbf{a}-\mathbf{b}\neq\mathbf{b}-\mathbf{a}$

(ii) $(\mathbf{a}-\mathbf{b})-\mathbf{c}\neq\mathbf{a}-(\mathbf{b}-\mathbf{c})$

(iii) Since any one side of a triangle is less than the sum and greater than the difference of the other two sides, so for any two vectors *a* and *b*, we have

(a) $|\mathbf{a}+\mathbf{b}|\leq|\mathbf{a}|+|\mathbf{b}|$

(b) $|\mathbf{a}+\mathbf{b}|\geq|\mathbf{a}|-|\mathbf{b}|$

(c) $|\mathbf{a}-\mathbf{b}|\leq|\mathbf{a}|+|\mathbf{b}|$

(d) $|\mathbf{a}-\mathbf{b}|\geq|\mathbf{a}|-|\mathbf{b}|$

(3) **Multiplication of a vector by a scalar :** If $\mathbf{a}$ is a vector and *m* is a scalar (*i.e.,* a real number) then $m\mathbf{a}$ is a vector whose magnitude is *m* times that of $\mathbf{a}$ and whose direction is the same as that of $\mathbf{a}$, if *m* is positive and opposite to that of $\mathbf{a}$, if *m* is negative.

$\therefore$ Magnitude of $m\mathbf{a}=|m\mathbf{a}|=m$ (magnitude of $\mathbf{a}$) $=m|\mathbf{a}|$

Again if $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ then $m\mathbf{a}=(ma_1)\mathbf{i}+(ma_2)\mathbf{j}+(ma_3)\mathbf{k}$

Properties of Multiplication of vectors by a scalar: The following are properties of multiplication of vectors by scalars, for vectors $\mathbf{a},\mathbf{b}$ and scalars *m*, *n*

(i) $m(-\mathbf{a})=(-m)\mathbf{a}=-(m\mathbf{a})$

(ii) $(-m)(-\mathbf{a})=m\mathbf{a}$

(iii) $m(n\mathbf{a})=(mn)\mathbf{a}=n(m\mathbf{a})$

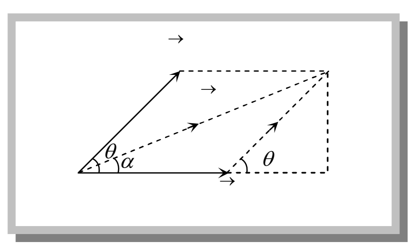
(iv) $(m+n)\mathbf{a}=m\mathbf{a}+n\mathbf{a}$

(v) $m(\mathbf{a}+\mathbf{b})=m\mathbf{a}+m\mathbf{b}$

(4) **Resultant of two forces**

$\vec{R}=\vec{P}+\vec{Q}$

$|\vec{R}|=R=\sqrt{P^2+Q^2+2PQ\cos\theta}$



Where $|\vec{P}| = P$, $|\vec{Q}| = Q$, $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

Deduction: When $|\vec{P}| = |\vec{Q}|$, *i.e.*, $P = Q$, $\tan\alpha = \dfrac{P\sin\theta}{P + P\cos\theta} = \dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2}$; $\therefore \alpha = \dfrac{\theta}{2}$

Hence, the angular bisector of two unit vectors $\mathbf{a}$ and $\mathbf{b}$ is along the vector sum $\mathbf{a} + \mathbf{b}$.

Important Tips

- *The internal bisector of the angle between any two vectors is along the vector sum of the corresponding unit vectors.*
- *The external bisector of the angle between two vectors is along the vector difference of the corresponding unit vectors.*