Linear Independence and Dependence of Vectors.

(1) **Linearly independent vectors:** A set of non-zero vectors $\mathbf{a}_1, \mathbf{a}_2, ... \mathbf{a}_n$ is said to be linearly independent, if $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + ... + x_n\mathbf{a}_n = \mathbf{0} \Rightarrow x_1 = x_2 = ... = x_n = 0$.

(2) **Linearly dependent vectors:** A set of vectors $\mathbf{a}_1, \mathbf{a}_2, ... \mathbf{a}_n$ is said to be linearly dependent if there exist scalars $x_1, x_2, ..., x_n$ not all zero such that $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + ... + x_n\mathbf{a}_n = \mathbf{0}$

Three vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ will be linearly dependent vectors *iff* $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$.

Properties of linearly independent and dependent vectors

(i) Two non-zero, non-collinear vectors are linearly independent.

(ii) Any two collinear vectors are linearly dependent.

(iii) Any three non-coplanar vectors are linearly independent.

(iv) Any three coplanar vectors are linearly dependent.

(v) Any four vectors in 3-dimensional space are linearly dependent.