Reduction of Cartesian form of the Equation of a line to Vector form and vice versa.

Cartesian to vector: Let the Cartesian equation of a line be $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ ......(i)

This is the equation of a line passing through the point $A(x_1, y_1, z_1)$ and having direction ratios a, b, c. In vector form this means that the line passes through point having position vector $\mathbf{a}=x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k}$ and is parallel to the vector $\mathbf{m}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$. Thus, the vector form of (i) is $\mathbf{r}=\mathbf{a}+\lambda\mathbf{m}$ or $\mathbf{r}=(x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k})+\lambda(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})$, where λ is a parameter.

Vector to cartesian: Let the vector equation of a line be $\mathbf{r}=\mathbf{a}+\lambda\mathbf{m}$ ......(ii)

Where $\mathbf{a}=x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k}, \mathbf{m}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ and λ is a parameter.

To reduce (ii) to Cartesian form we put $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and equate the coefficients of **i**, **j** and **k** as discussed below.

Putting $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}, \mathbf{a}=x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k}$ and $\mathbf{m}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ in (ii), we obtain

$x\mathbf{i}+y\mathbf{j}+z\mathbf{k}=(x_1\mathbf{i}+y_1\mathbf{j}+z_1\mathbf{k})+\lambda(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})$

Equating coefficients of **i**, **j** and **k**, we get $x=x_1+a\lambda, y=y_1+b\lambda, z=z_1+c\lambda$ or

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\lambda$$