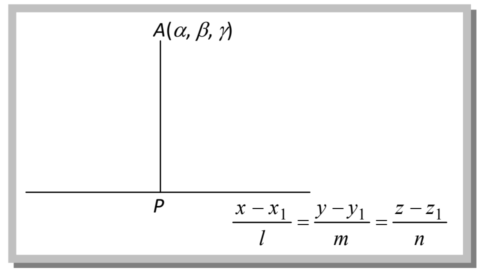
Foot of perpendicular from a point A(α, β, γ) to the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$.

(1) **Cartesian form**

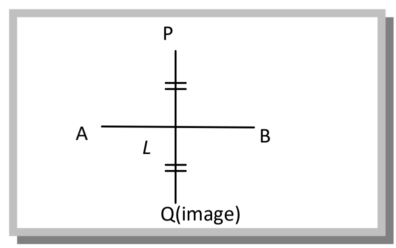
Foot of perpendicular from a point A(α, β, γ) to the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ **:** If P be the foot of perpendicular, then P is $(lr+x_1, mr+y_1, nr+z_1)$. Find the direction ratios of AP and apply the condition of perpendicularity of AP and the given line. This will give the value of r and hence the point P which is foot of perpendicular.



Length and equation of perpendicular: The length of the perpendicular is the distance APand its equation is the line joining two known points A and P.

Note: The length of the perpendicular is the perpendicular distance of given point from that line.

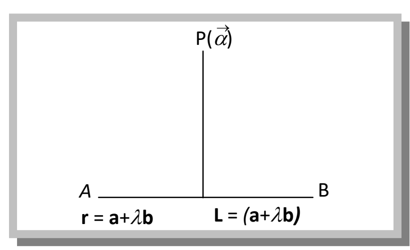
Reflection or image of a point in a straight line: If the perpendicular PL from point P on the given line be produced to Q such that PL = QL, then Q is known as the image or reflection of P in the given line. Also, L is the foot of the perpendicular or the projection of P on the line.



(2) **Vector form**

Perpendicular distance of a point from a line: Let L is the foot of perpendicular drawn from $P(\vec{\alpha})$ on the line $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$. Since **r** denotes the position vector of any point on the line $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$. So, let the position vector of L be $\mathbf{a}+\lambda\mathbf{b}$.

Then $\overrightarrow{PL}=\mathbf{a}-\vec{\alpha}+\lambda\mathbf{b}=(\mathbf{a}-\vec{\alpha})-\left(\frac{(\mathbf{a}-\vec{\alpha})\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b}$



The length PL, is the magnitude of $\overrightarrow{PL}$, and required length of perpendicular.

Image of a point in a straight line : Let $Q(\vec{\beta})$ is the image of P in $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$

Then, $\vec{\beta} = 2\mathbf{a} - \left(\frac{2(\mathbf{a} - \vec{\alpha}).\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b}.\alpha$