Definition of plane and its equations.

If point P(x, y, z) moves according to certain rule, then it may lie in a 3-D region on a surface or on a line or it may simply be a point. Whatever we get, as the region of P after applying the rule, is called locus of P. Let us discuss about the plane or curved surface. If Q be any other point on it's locus and all points of the straight line PQ lie on it, it is a plane. In other words if the straight line PQ, however small and in whatever direction it may be, lies completely on the locus, it is a plane, otherwise any curved surface.

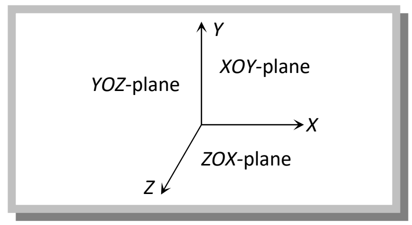
(1) **General equation of plane:** Every equation of first degree of the form $Ax + By + Cz + D = 0$ represents the equation of a plane. The coefficients of x, y and z i.e. A, B, C are the direction ratios of the normal to the plane.

(2) **Equation of co-ordinate planes**

XOY-plane: z = 0

YOZ -plane: x = 0

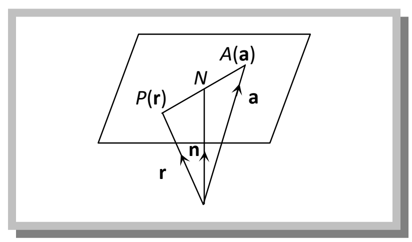
ZOX-plane: y = 0



(3) **Vector equation of plane**

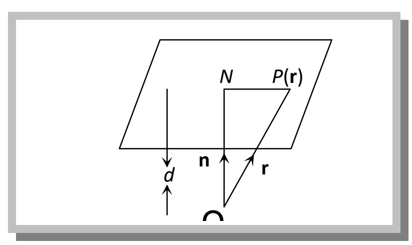
(i) Vector equation of a plane through the point $A(\mathbf{a})$ and perpendicular to the vector **n** is

$(\mathbf{r}-\mathbf{a}).\mathbf{n} = 0$ or $\mathbf{r}.\mathbf{n} = \mathbf{a}.\mathbf{n}$



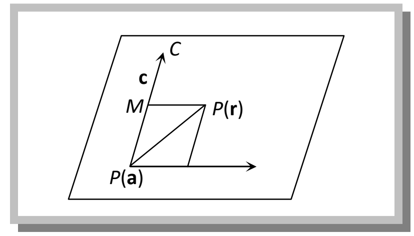
Note: The above equation can also be written as $\mathbf{r}.\mathbf{n} = d$, where $d = \mathbf{a}.\mathbf{n}$. This is known as the scalar product form of a plane.

(4) **Normal form:** Vector equation of a plane normal to unit vector $\hat{\mathbf{n}}$ and at a distance d from the origin is $\mathbf{r}.\hat{\mathbf{n}} = d$.



Note: If **n** is not a unit vector, then to reduce the equation $\mathbf{r}.\mathbf{n} = d$ to normal form we divide both sides by |**n**| to obtain $\mathbf{r} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{d}{|\mathbf{n}|}$ or $\mathbf{r}.\hat{\mathbf{n}} = \frac{d}{|\mathbf{n}|}$.

(5) **Equation of a plane passing through a given point and parallel to two given vectors:** The equation of the plane passing through a point having position vector **a** and parallel to **b** and **c** is $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where λ and μ are scalars.



(6) **Equation of plane in various forms**

(i) **Intercept form**: If the plane cuts the intercepts of length a, b, c on co-ordinate axes, then its equation is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(ii) **Normal form:** Normal form of the equation of plane is $lx + my + nz = p$,

where l, m, n are the d.c.'s of the normal to the plane and p is the length of perpendicular from the origin.

(7) **Equation of plane in particular cases**

(i) Equation of plane through the origin is given by $Ax + By + Cz = 0$.

i.e. if D = 0, then the plane passes through the origin.

(8) **Equation of plane parallel to co-ordinate planes or perpendicular to co-ordinate axes**

(i) Equation of plane parallel to YOZ-plane (or perpendicular to x-axis) and at a distance 'a' from it is x = a.

(ii) Equation of plane parallel to ZOX-plane (or perpendicular to y-axis) and at a distance 'b' from it is y = b.

(iii) Equation of plane parallel to XOY-plane (or perpendicular to z-axis) and at a distance 'c' from it is z = c.

Important Tips

- ☞ Any plane perpendicular to co-ordinate axis is evidently parallel to co-ordinate plane and vice versa.
- ☞ A unit vector perpendicular to the plane containing three points A, B, C is $\frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|}$.

(9) **Equation of plane perpendicular to co-ordinate planes or parallel to co-ordinate axes**

(i) Equation of plane perpendicular to YOZ-plane or parallel to x-axis is $By + Cz + D = 0$.

(ii) Equation of plane perpendicular to ZOX-plane or parallel to y axis is $Ax + Cz + D = 0$.

(iii) Equation of plane perpendicular to XOY-plane or parallel to z-axis is $Ax + By + D = 0$.

(10) **Equation of plane passing through the intersection of two planes**

(i) **Cartesian form:**Equation of plane through the intersection of two planes $P = a_1x + b_1y + c_1z + d_1 = 0$ and $Q = a_2x + b_2y + c_2z + d_2 = 0$ is $P + \lambda Q = 0$, where λ is the parameter.

(ii) **Vector form:**The equation of any plane through the intersection of planes $\mathbf{r}.\mathbf{n}_1 = d_1$ and $\mathbf{r}.\mathbf{n}_2 = d_2$ is $\mathbf{r}.(\mathbf{n}_1 + \lambda\mathbf{n}_2) = d_1 + \lambda d_2$, where λ is an arbitrary constant.

(11) **Equation of plane parallel to a given plane**

(i) **Cartesian form:**Plane parallel to a given plane $ax + by + cz + d = 0$ is $ax + by + cz + d' = 0$, i.e. only constant term is changed.

(ii) **Vector form:**Since parallel planes have the common normal, therefore equation of plane parallel to plane $\mathbf{r}.\mathbf{n} = d_1$ is $\mathbf{r}.\mathbf{n} = d_2$, where d_2 is a constant determined by the given condition.