Foot of perpendicular from a point A(α, β, γ) to a given plane ax + by + cz + d = 0.

If AP be the perpendicular from A to the given plane, then it is parallel to the normal, so that its equation is

$$\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}=r \qquad \text{(say)}$$

Any point P on it is $(ar+\alpha, br+\beta, cr+\gamma)$. It lies on the given plane and we find the value of r and hence the point P.

(1) **Perpendicular distance**

(i) **Cartesian form :**The length of the perpendicular from the point $P(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$ is $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$.

Note: The distance between two parallel planes is the algebraic difference of perpendicular distances on the planes from origin.

Distance between two parallel planes $Ax+By+Cz+D_1=0$ and $Ax+By+Cz+D_2=0$ is $\frac{D_2 \sim D_1}{\sqrt{A^2+B^2+C^2}}$.

(ii) **Vector form:**The perpendicular distance of a point having position vector **a** from the plane $\mathbf{r}.\mathbf{n}=d$ is given by $p=\frac{|\mathbf{a}.\mathbf{n}-d|}{|\mathbf{n}|}$

(2) **Position of two point's w.r.t. a plane:**Two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ lie on the same or opposite sides of a plane $ax+by+cz+d=0$ according to $ax_1+by_1+cz_1+d$ and $ax_2+by_2+cz_2+d$ are of same or opposite signs. The plane divides the line joining the points P and Q externally or internally according to P and Q are lying on same or opposite sides of the plane.