Equation of planes bisecting angle between two given planes.

(1) **Cartesian form:**Equations of planes bisecting angles between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d = 0$ are

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{(a_1^2 + b_1^2 + c_1^2)}} = \pm \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{(a_2^2 + b_2^2 + c_2^2)}}.$$

Note: If angle between bisector plane and one of the plane is less than 45°, then it is acute angle bisector otherwise it is obtuse angle bisector.

If $a_1a_2 + b_1b_2 + c_1c_2$ is negative, then origin lies in the acute angle between the given planes provided d_1 and d_2 are of same sign and if $a_1a_2 + b_1b_2 + c_1c_2$ is positive, then origin lies in the obtuse angle between the given planes.

(2) **Vector form:**The equation of the planes bisecting the angles between the planes $\mathbf{r}_1.\mathbf{n}_1 = d_1$ and $\mathbf{r}_2.\mathbf{n}_2 = d_2$ are $\frac{|\mathbf{r}.\mathbf{n}_1 - d_1|}{|\mathbf{n}_1|} = \frac{|\mathbf{r}.\mathbf{n}_2 - d_2|}{|\mathbf{n}_2|}$ or $\frac{\mathbf{r}.\mathbf{n}_1 - d_1}{|\mathbf{n}_1|} = \pm \frac{\mathbf{r}.\mathbf{n}_2 - d_2}{|\mathbf{n}_2|}$ or

$$\mathbf{r}.(\hat{\mathbf{n}}_1 \pm \hat{\mathbf{n}}_2) = \frac{d_1}{|\mathbf{n}_1|} \pm \frac{d_2}{|\mathbf{n}_2|}.$$