Coplanar lines.

Lines are said to be coplanar if they lie in the same plane or a plane can be made to pass through them.

(1) **Condition for the lines to be coplanar**

(i) **Cartesian form:** If the lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar

Then $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$.

The equation of the plane containing them is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$ or $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$.

(ii) **Vector form:** If the lines $\mathbf{r}=\mathbf{a}_1+\lambda\mathbf{b}_1$ and $\mathbf{r}=\mathbf{a}_2+\lambda\mathbf{b}_2$ are coplanar, then $[\mathbf{a}_1\mathbf{b}_1\mathbf{b}_2]=[\mathbf{a}_2\mathbf{b}_1\mathbf{b}_2]$ and the equation of the plane containing them is $[\mathbf{r}\,\mathbf{b}_1\,\mathbf{b}_2]=[\mathbf{a}_1\,\mathbf{b}_1\,\mathbf{b}_2]$ or $[\mathbf{r}\,\mathbf{b}_1\,\mathbf{b}_2]=[\mathbf{a}_2\,\mathbf{b}_1\,\mathbf{b}_2]$.

Note: Every pair of parallel lines is coplanar.

Two coplanar lines are either parallel or intersecting.

The three sides of a triangle are coplanar.

Important Tips

- **Division by plane :** The ratio in which the line segment PQ, joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2), is divided by plane $ax+by+cz+d=0$ is $=-\left(\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}\right)$.
- **Division by co-ordinate planes :** The ratio in which the line segment PQ, joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) is divided by co-ordinate planes are as follows :

(i) By yz-plane : $-x_1/x_2$ (ii) By zx-plane : $-y_1/y_2$ (ii) By xy-plane : $-z_1/z_2$