Angle between line and plane.

(1) **Cartesian form:** The angle θ between the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$, and the plane $ax+by+cz+d=0$, is given by $\sin\theta=\frac{al+bm+cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{(l^2+m^2+n^2)}}$.

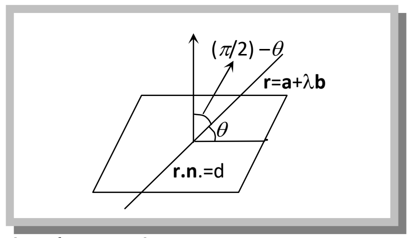
(i) The line is perpendicular to the plane if and only if $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$.

(ii) The line is parallel to the plane if and only if $al+bm+cn=0$.

(iii) The line lies in the plane if and only if $al+bm+cn=0$ and $a\alpha+b\beta+c\gamma+d=0$.

(2) **Vector form:** If θ is the angle between a line $\mathbf{r}=(\mathbf{a}+\lambda\mathbf{b})$ and the plane $\mathbf{r}.\mathbf{n}=d$, then $\sin\theta=\frac{\mathbf{b}.\mathbf{n}}{|\mathbf{b}||\mathbf{n}|}$.

(i) **Condition of perpendicularity:** If the line is perpendicular to the plane, then it is parallel to the normal to the plane. Therefore **b** and **n** are parallel.
So, $\mathbf{b}\times\mathbf{n}=0$ or **b** = λ**n** for some scalar λ.



(ii) **Condition of parallelism:** If the line is parallel to the plane, then it is perpendicular to the normal to the plane. Therefore **b** and **n** are perpendicular. So, **b**.**n** = 0.

(iii) If the line $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$ lies in the plane **r**.**n** = d, then (i) **b**.**n** = 0 and (ii) **a**.**n** = d.