

## General equation of sphere.

The general equation of a sphere is  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$  with centre  $(-u, -v, -w)$

i.e.  $(-1/2)$  coeff. of  $x$ ,  $(-1/2)$  coeff. of  $y$ ,  $(-1/2)$  coeff. of  $z$  and, radius  $= \sqrt{u^2 + v^2 + w^2 - d}$

From the above equation, we note the following characteristics of the equation of a sphere :

- (i) It is a second degree equation in  $x, y, z$ ;
- (ii) The coefficients of  $x^2, y^2, z^2$  are all equal;
- (iii) The terms containing the products  $xy, yz$  and  $zx$  are absent.

**Note:** The equation  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$  represents,

- (i) A real sphere, if  $u^2 + v^2 + w^2 - d > 0$ .
- (ii) A point sphere, if  $u^2 + v^2 + w^2 - d = 0$ .
- (iii) An imaginary sphere, if  $u^2 + v^2 + w^2 - d < 0$ .

### Important Tips

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- ☞ If  $u^2 + v^2 + w^2 - d < 0$ , then the radius of sphere is imaginary, whereas the centre is real. Such a sphere is called "pseudo-sphere" or a "virtual sphere."
- ☞ The equation of the sphere contains four unknown constants  $u, v, w$  and  $d$  and therefore a sphere can be found to satisfy four conditions.