Condition of tangency of a plane to a sphere.

A plane touches a given sphere if the perpendicular distance from the centre of the sphere to the plane is equal to the radius of the sphere.

(1) **Cartesian form:** The plane $lx + my + nz = p$ touches the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, if $(ul + vm + wn - p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$

(2) **Vector form:** The plane $\mathbf{r}.\mathbf{n} = d$ touches the sphere $|\mathbf{r} - \mathbf{a}| = R$ if $\dfrac{|\mathbf{a}.\mathbf{n} - d|}{|\mathbf{n}|} = R$.

Important Tips

- ☞ Two spheres S_1 and S_2 with centres C_1 and C_2 and radii r_1 and r_2 respectively

(i) Do not meet and lies farther apart iff $|C_1C_2| > r_1 + r_2$

(ii) Touch internally iff $|C_1C_2| = |r_1 - r_2|$

(iii) Touch externally iff $|C_1C_2| = r_1 + r_2$

(iv) Cut in a circle iff $|r_1 - r_2| < |C_1C_2| < r_1 + r_2$

(v) One lies within the other if $|C_1C_2| < |r_1 - r_2|$.

When two spheres touch each other the common tangent plane is $S_1 - S_2 = 0$ and when they cut in a circle, the plane of the circle is $S_1 - S_2 = 0$; coefficients of x^2, y^2, z^2 being unity in both the cases.

- ☞ Let p be the length of perpendicular drawn from the centre of the sphere $x^2 + y^2 + z^2 = r^2$ to the plane $Ax + By + Cz + D = 0$, then

(i) The plane cuts the sphere in a circle iff p < r and in this case, the radius of circle is $\sqrt{r^2 - p^2}$.

(ii) The plane touches the sphere iff $p = r$.

(iii) The plane does not meet the sphere iff p > r.

- ☞ **Equation of concentric sphere :** Any sphere concentric with the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + \lambda = 0$, where λ is some real which makes it a sphere.