Intersection of straight line and a sphere.

Let the equations of the sphere and the straight line be $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ .....(i)

And $$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r \quad \text{(say)} \qquad .....(ii)$$

Any point on the line (ii) is $(\alpha + lr, \beta + mr, \gamma + nr)$.

If this point lies on the sphere (i) then we have,

$(\alpha + lr)^2 + (\beta + mr)^2 + (\gamma + nr)^2 + 2u(\alpha + lr) + 2v(\beta + mr) + 2w(\gamma + nr) + d = 0$

or, $r^2[l^2 + m^2 + n^2] + 2r[l(u+\alpha) + m(v+\beta)] + n(w+\gamma)] + (\alpha^2 + \beta^2 + \gamma^2 + 2u\alpha + 2v\beta + 2w\gamma + d) = 0$ ....(iii)

This is a quadratic equation in r and so gives two values of r and therefore the line (ii) meets the sphere (i) in two points which may be real, coincident and imaginary, according as root of (iii) are so.

Note: If l, m, n are the actual d.c.'s of the line, then $l^2 + m^2 + n^2 = 1$ and then the equation (iii) can be simplified.