

Angle of intersection of two spheres.

The angle of intersection of two spheres is the angle between the tangent planes to them at their point of intersection. As the radii of the spheres at this common point are normal to the tangent planes so this angle is also equal to the angle between the radii of the spheres at their point of intersection.

If the angle of intersection of two spheres is a right angle, the spheres are said to be orthogonal.

Condition for orthogonality of two spheres

Let the equation of the two spheres be

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad \dots(i)$$

$$\text{and } x^2 + y^2 + z^2 + 2u'x + 2v'y + 2w'z + d' = 0 \quad \dots(ii)$$

If the sphere (i) and (ii) cut orthogonally, then $2uu' + 2vv' + 2ww' = d + d'$, which is the required condition.

Note: If the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ cut orthogonally, then $d = a^2$.

Two spheres of radii r_1 and r_2 cut orthogonally, then the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$