

Triangle.

(1) Co-ordinates of the centroid

(i) If $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) are the vertices of a triangle, then co-ordinates of its centroid are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.

(ii) If $(x_r, y_r, z_r); r = 1, 2, 3, 4$, are vertices of a tetrahedron, then co-ordinates of its centroid are $\left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4}\right)$.

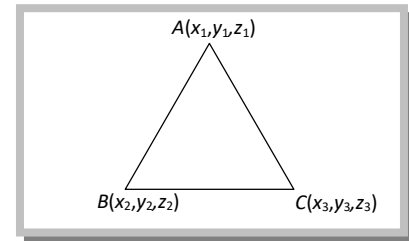
(iii) If $G(\alpha, \beta, \gamma)$ is the centroid of ΔABC , where A is (x_1, y_1, z_1) , B is (x_2, y_2, z_2) , then C is $(3\alpha - x_1 - x_2, 3\beta - y_1 - y_2, 3\gamma - z_1 - z_2)$.

(2) **Area of triangle:** Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be the vertices of a triangle, then

$$\Delta_x = \frac{1}{2} \begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}, \Delta_y = \frac{1}{2} \begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix}, \Delta_z = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, area of ΔABC is given by the relation $\Delta = \sqrt{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}$.

$$\text{Also, } \Delta = \frac{1}{2} | \vec{AB} \times \vec{AC} | = \frac{1}{2} \left| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} \right|$$



(3) **Condition of collinearity:** Points $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear

$$\text{If } \frac{x_1 - x_2}{x_2 - x_3} = \frac{y_1 - y_2}{y_2 - y_3} = \frac{z_1 - z_2}{z_2 - z_3}$$