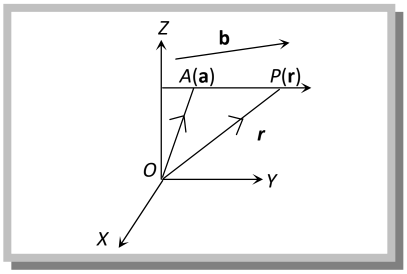
Straight line in Space.

Every equation of the first degree represents a plane. Two equations of the first degree are satisfied by the co-ordinates of every point on the line of intersection of the planes represented by them. Therefore, the two equations together represent that line. Therefore $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ together represent a straight line.

(1) **Equation of a line passing through a given point**

(i) **Cartesian form or symmetrical form:** Cartesian equation of a straight line passing through a fixed point (x_1, y_1, z_1) and having direction ratios a, b, c is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$.

(ii) **Vector form:** Vector equation of a straight line passing through a fixed point with position vector **a** and parallel to a given vector **b** is $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.



Important Tips

☞ The parametric equations of the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ are $x = x_1 + a\lambda, y = y_1 + b\lambda, z = z_1 + c\lambda$, where λ is the parameter.

☞ The co-ordinates of any point on the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ are $(x_1 + a\lambda, y_1 + b\lambda, z_1 + c\lambda)$, where $\lambda \in$ R.

☞ Since the direction cosines of a line are also direction ratios, therefore equation of a line passing through (x_1, y_1, z_1) and having direction cosines l, m, n is $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$.

☞ Since x, y and z-axes pass through the origin and have direction cosines 1, 0, 0; 0, 1, 0 and 0, 0, 1 respectively. Therefore, the equations are x-axis : $\frac{x - 0}{1} = \frac{y - 0}{0} = \frac{z - 0}{0}$ or y = 0 and z = 0.

y-axis : $\frac{x-0}{0}=\frac{y-0}{1}=\frac{z-0}{0}$ or x = 0 and z = 0; z-axis : $\frac{x-0}{0}=\frac{y-0}{0}=\frac{z-0}{1}$ or x = 0 and y = 0.

☞ In the symmetrical form of equation of a line, the coefficients of x, y, z are unity.