

Formulae for the Trigonometric Ratios of Sum and Differences of Three Angles.

$$(1) \sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$$

$$\text{or } \sin(A + B + C) = \cos A \cos B \cos C (\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C)$$

$$(2) \cos(A + B + C) = \cos A \cos B \cos C - \sin A \sin B \cos C - \sin A \cos B \sin C - \cos A \sin B \sin C$$

$$\cos(A + B + C) = \cos A \cos B \cos C (1 - \tan A \tan B - \tan B \tan C - \tan C \tan A)$$

$$(3) \tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

$$(4) \cot(A + B + C) = \frac{\cot A \cot B \cot C - \cot A - \cot B - \cot C}{\cot A \cot B + \cot B \cot C + \cot C \cot A - 1}$$

In general:

$$(5) \sin(A_1 + A_2 + \dots + A_n) = \cos A_1 \cos A_2 \dots \cos A_n (S_1 - S_3 + S_5 - S_7 + \dots)$$

$$(6) \cos(A_1 + A_2 + \dots + A_n) = \cos A_1 \cos A_2 \dots \cos A_n (1 - S_2 + S_4 - S_6 \dots)$$

$$(7) \tan(A_1 + A_2 + \dots + A_n) = \frac{S_1 - S_3 + S_5 - S_7 + \dots}{1 - S_2 + S_4 - S_6 + \dots}$$

Where; $S_1 = \tan A_1 + \tan A_2 + \dots + \tan A_n$ = The sum of the tangents of the separate angles.

$S_2 = \tan A_1 \tan A_2 + \tan A_1 \tan A_3 + \dots$ = The sum of the tangents taken two at a time.

$S_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + \dots$ = Sum of tangents three at a time, and so on.

If $A_1 = A_2 = \dots = A_n = A$, then $S_1 = n \tan A$, $S_2 = {}^nC_2 \tan^2 A$, $S_3 = {}^nC_3 \tan^3 A$, ...

$$(8) \sin nA = \cos^n A ({}^nC_1 \tan A - {}^nC_3 \tan^3 A + {}^nC_5 \tan^5 A - \dots)$$

$$(9) \cos nA = \cos^n A (1 - {}^nC_2 \tan^2 A + {}^nC_4 \tan^4 A - \dots)$$

$$(10) \tan nA = \frac{"C_1 \tan A - "C_3 \tan^3 A + "C_5 \tan^5 A - \dots}{1 - "C_2 \tan^2 A + "C_4 \tan^4 A - "C_6 \tan^6 A + \dots}$$

(11)

$$\sin nA + \cos nA = \cos^n A (1 + "C_1 \tan A - "C_2 \tan^2 A + "C_3 \tan^3 A - "C_4 \tan^4 A + "C_5 \tan^5 A - "C_6 \tan^6 A - \dots)$$

(12)

$$\sin nA - \cos nA = \cos^n A (-1 + "C_1 \tan A + "C_2 \tan^2 A - "C_3 \tan^3 A - "C_4 \tan^4 A + "C_5 \tan^5 A + "C_6 \tan^6 A \dots)$$

$$(13) \sin(\alpha) + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots + \sin(\alpha + (n-1)\beta) = \frac{\sin\{\alpha + (n-1)(\beta/2)\} \cdot \sin(n\beta/2)}{\sin(\beta/2)}$$

$$(14) \cos(\alpha) + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots + \cos(\alpha + (n-1)\beta) = \frac{\cos\left\{\alpha + (n-1)\left(\frac{\beta}{2}\right)\right\} \cdot \sin\left\{n\left(\frac{\beta}{2}\right)\right\}}{\sin\left(\frac{\beta}{2}\right)}$$