

Formulae to Transform the Product into Sum or Difference.

$$(1) 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$(2) 2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$(3) 2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$(4) 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

Let $A + B = C$ and $A - B = D$

$$\text{Then, } A = \frac{C + D}{2} \text{ and } B = \frac{C - D}{2}$$

Therefore, we find out the formulae to transform the sum or difference into product.

$$(5) \sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$(6) \sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$$

$$(7) \cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$(8) \cos C - \cos D = 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2} = -2 \sin \frac{C + D}{2} \sin \frac{C - D}{2}$$

Important Tips

$$\Rightarrow \sin(60^\circ - \theta) \cdot \sin \theta \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$$

$$\Rightarrow \cos(60 - \theta) \cdot \cos \theta \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$$

$$\Rightarrow \tan(60^\circ - \theta) \cdot \tan \theta \tan(60^\circ + \theta) = \tan 3\theta$$

$$\Rightarrow \cos A \cdot \cos 2A \cdot \cos 2^2 A \cdot \cos 2^3 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}, \text{ if } A = n\pi$$

$$= 1, \text{ if } A = 2n\pi$$

$$= 1, \text{ if } A = (2n + 1)\pi$$