

Trigonometric Ratio of Multiple of an Angle.

$$(1) \sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(2) \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}; \text{ where } A \neq (2n+1)\frac{\pi}{4}.$$

$$(3) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$(4) \sin 3A = 3 \sin A - 4 \sin^3 A = 4 \sin(60^\circ - A) \cdot \sin A \cdot \sin(60^\circ + A)$$

$$(5) \cos 3A = 4 \cos^3 A - 3 \cos A = 4 \cos(60^\circ - A) \cdot \cos A \cdot \cos(60^\circ + A)$$

$$(6) \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} = \tan(60^\circ - A) \cdot \tan A \cdot \tan(60^\circ + A), \text{ where } A \neq n\pi + \pi/6$$

$$(7) \sin 4\theta = 4 \sin \theta \cdot \cos^3 \theta - 4 \cos \theta \sin^3 \theta \quad (8) \cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$$

$$(9) \tan 4\theta = \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta} \quad (10) \sin 5A = 16 \sin^5 A - 20 \sin^3 A + 5 \sin A$$

$$(11) \cos 5A = 16 \cos^5 A - 20 \cos^3 A + 5 \cos A$$