Trigonometric Ratio of Sub-multiple of an Angle.

(1) $\left|\sin\frac{A}{2}+\cos\frac{A}{2}\right|=\sqrt{1+\sin A}$ or $\sin\frac{A}{2}+\cos\frac{A}{2}=\pm\sqrt{1+\sin A}$ i.e.,

$$\begin{cases}+, \text{If } 2n\pi-\pi/4\le A/2\le 2n\pi+\dfrac{3\pi}{4}\\ -, \text{otherwise}\end{cases}$$

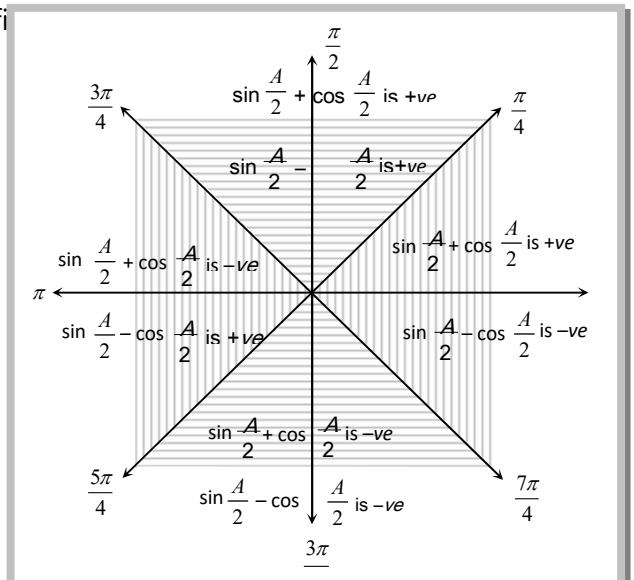
(2) $\left|\sin\frac{A}{2}-\cos\frac{A}{2}\right|=\sqrt{1-\sin A}$ or $(\sin\frac{A}{2}-\cos\frac{A}{2})=\pm\sqrt{1-\sin A}$ i.e.,

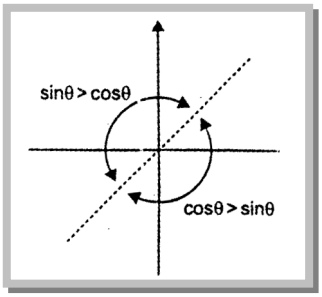
$$\begin{cases}+, \text{If } 2n\pi+\pi/4\le A/2\le 2n\pi+\dfrac{5\pi}{4}\\ -, \text{otherwise}\end{cases}$$

(3) (i) $\tan\frac{A}{2}=\frac{\pm\sqrt{\tan^2 A+1}-1}{\tan A}=\pm\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{1-\cos A}{\sin A}$, where $A\neq(2n+1)\pi$

(ii) $\cot\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{1-\cos A}}=\frac{1+\cos A}{\sin A}$, where $A\neq 2n\pi$

The ambiguities of signs are removed by locating the quadrants in which $\frac{A}{2}$ lies or you can follow the following fi





(4) $\tan^2 \frac{A}{2} = \frac{1-\cos A}{1+\cos A}$; where $A \neq (2n+1)\pi$

(5) $\cot^2 \frac{A}{2} = \frac{1+\cos A}{1-\cos A}$; where $A \neq 2n\pi$

Important Tips

- Any formula that gives the value of $\sin \frac{A}{2}$ in terms of sin A shall also give the value of sine of $\frac{n\pi + (-1)^n A}{2}$.
- Any formula that gives the value of $\cos \frac{A}{2}$ in terms of cos A shall also give the value of cos of $\frac{2n\pi \pm A}{2}$.
- Any formula that gives the value of $\tan \frac{A}{2}$ in terms of tan A shall also give the value of tan of $\frac{n\pi \pm A}{2}$.