Maximum and Minimum Value of a cosθ + b sinθ.

Let $a = r\cos\alpha$ ....(i) and $b = r\sin\alpha$ ....(ii)

Squaring and adding (i) and (ii), then $a^2 + b^2 = r^2$ or, $r = \sqrt{a^2 + b^2}$

$\therefore \quad a\sin\theta + b\cos\theta = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = r\sin(\theta + \alpha)$

But $-1 \le \sin\theta < 1$ So, $-1 \le \sin(\theta + \alpha) \le 1$; Then $-r \le r\sin(\theta + \alpha) \le r$

Hence, $-\sqrt{a^2 + b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2 + b^2}$

Then the greatest and least values of $a\sin\theta + b\cos\theta$ are respectively $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$.

Note: $\sin^2 x + cosec^2 x \ge 2,$ for every real x.

$\cos^2 x + \sec^2 x \ge 2,$ for every real x.

$\tan^2 x + \cot^2 x \ge 2$, for every real x.

Important Tips

Use of Σ(Sigma) and $\prod$(Pie) notation

- $\sin(A + B + C) = \Sigma \sin A \cos B \cos C - \Pi \sin A$, $\cos(A + B + C) = \Pi \cos A - \Sigma \cos A \sin B \sin C$,

  $\tan(A + B + C) = \dfrac{\Sigma \tan A - \Pi \tan A}{1 - \Sigma \tan A \tan B}$. ($\because$ Σ denotes summation)

- $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... n$ terms ($\because$ Π denotes product)

  $= \dfrac{\sin[\alpha + (n-1)\beta/2]\sin[n\beta/2]}{\sin(\beta/2)}$ or $\sum_{r=1}^{n} \sin(A + \overline{r-1}B) = \dfrac{\sin\left(A + \dfrac{n-1}{2}B\right)\sin\dfrac{nB}{2}}{\sin\dfrac{B}{2}}$.

- $\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + ... n$ terms $= \dfrac{\cos[\alpha + (n-1)\beta/2]\sin[n\beta/2]}{\sin[\beta/2]}$ or

$$\sum_{r=1}^{n} \cos(A + \overline{r-1}B) = \dfrac{\cos\left(A + \dfrac{n-1}{2}B\right)\sin\dfrac{nB}{2}}{\sin\dfrac{B}{2}}.$$

- $\sin A/2 \pm \cos A/2 = \sqrt{2}\sin[\pi/4 \pm A] = \sqrt{2}\cos[A \mp \pi/4]$.
- $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha + \beta + \gamma) = 4\cos\dfrac{\alpha + \beta}{2}\cos\dfrac{\beta + \gamma}{2}\cos\dfrac{\gamma + \alpha}{2}$.
- $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha + \beta + \gamma) = 4\sin\dfrac{\alpha + \beta}{2}\sin\dfrac{\beta + \gamma}{2}\sin\dfrac{\gamma + \alpha}{2}$.
- $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.