Conditional Trigonometrical Identities.

We have certain trigonometric identities. Like, $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$ etc.
Such identities are identities in the sense that they hold for all value of the angles which satisfy the given condition among them and they are called conditional identities.
If A, B, C denote the angles of a triangle ABC, then the relation A + B + C = π enables us to establish many important identities involving trigonometric ratios of these angles.

(1) If A + B + C = π, then A + B = π – C, B + C = π – A and C + A = π – B.

(2) If A + B + C = π, then $\sin(A+B) = \sin(\pi - C) = \sin C$

Similarly, $\sin(B+C) = \sin(\pi - A) = \sin A$ and $\sin(C+A) = \sin(\pi - B) = \sin B$

(3) If $A + B + C = \pi$, then $\cos(A+B) = \cos(\pi - C) = -\cos C$

Similarly, $\cos(B+C) = \cos(\pi - A) = -\cos A$ and $\cos(C+A) = \cos(\pi - B) = -\cos B$

(4) If A + B + C = π, then $\tan(A+B) = \tan(\pi - C) = -\tan C$

Similarly, $\tan(B+C) = \tan(\pi - A) = -\tan A$ and $\tan(C+A) = \tan(\pi - B) = -\tan B$

(5) If $A + B + C = \pi$, then $\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$ and $\frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2}$ and $\frac{C+A}{2} = \frac{\pi}{2} - \frac{B}{2}$

$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\left(\frac{C}{2}\right)$, $\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = \sin\left(\frac{C}{2}\right)$,

$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\left(\frac{C}{2}\right)$

All problems on conditional identities are broadly divided into the following three types

1. **Identities involving sine and cosine of the multiple or sub-multiple of the angles involved**

Working Method

Step (i)**:** Use $C \pm D$ formulae.

Step (ii)**:** Use the given relation (A + B + C = π) in the expression obtained in step-(i) such that a factor can be taken common after using multiple angles formulae in the remaining term.

Step (iii)**:** Take the common factor outside.

Step (iv)**:** Again use the given relation (A + B + C =π) within the bracket in such a manner so that we can apply $C \pm D$ formulae.

Step (v)**:** Find the result according to the given options.

2. **Identities involving squares of sine and cosine of multiple or sub-multiples of the angles involved**

Working Method

Step (i)**:**Arrange the terms of the identity such that either $\sin^2 A - \sin^2 B = \sin(A+B).\sin(A-B)$ or $\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$ can be used.

Step (ii)**:** Take the common factor outside.

Step (iii)**:** Use the given relation $(A+B+C=\pi)$ within the bracket in such a manner so that we can apply $C \pm D$ formulae.

Step (iv)**:** Find the result according to the given options.

3. **Identities for tangent and cotangent of the angles**

Working Method

Step (i)**:** Express the sum of the two angles in terms of third angle by using the given relation $(A+B+C=\pi)$.

Step (ii)**:** Taking tangent or cotangent of the angles of both the sides.

Step (iii)**:** Use sum and difference formulae in the left hand side.

Step (iv)**:** Use cross multiplication in the expression obtained in the step (iii).

Step (v)**:** Arrange the terms as per the result required.

Important Tips

☞ **Method of componendo and dividendo**

If $\frac{p}{q} = \frac{a}{b}$, then by componendo and dividendo

We can write $\frac{p+q}{p-q} = \frac{a+b}{a-b}$ or $\frac{q+p}{q-p} = \frac{b+a}{b-a}$ or $\frac{p-q}{p+q} = \frac{a-b}{a+b}$ or $\frac{q-p}{q+p} = \frac{b-a}{b+a}$.