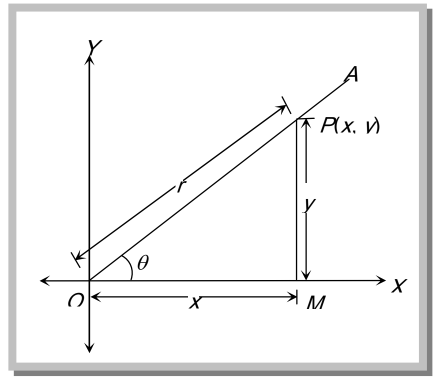
Trigonometrical Ratios or Functions.

In the right angled triangle OMP, we have base = OM = x, perpendicular =PM = y and hypotenuse = OP =r. We define the following trigonometric ratio which are also known as trigonometric function.

$\sin\theta = \dfrac{\text{Perpendicu lar}}{\text{Hypotenues}} = \dfrac{y}{r}$ $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenues}} = \dfrac{x}{r}$

$\tan\theta = \dfrac{\text{Perpendicu lar}}{\text{Base}} = \dfrac{y}{x}$ $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicu lar}} = \dfrac{x}{y}$,

$\sec\theta = \dfrac{\text{Hypotenues}}{\text{Base}} = \dfrac{r}{x}$ $\text{cosec}\,\theta = \dfrac{\text{Hypotenues}}{\text{Perpendicu lar}} = \dfrac{r}{y}$



(1) **Relation between trigonometric ratios (function)**

(i) $\sin\theta . co\sec\theta = 1$

(ii) $\tan\theta . \cot\theta = 1$

(iii) $\cos\theta . \sec\theta = 1$

(iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(v) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(2) **Fundamental trigonometric identities**

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

Important Tips

☞ If $x = \sec\theta + \tan\theta$, then $\frac{1}{x} = \sec\theta - \tan\theta$.

☞ If $x = \text{coesc}\,\theta + \cot\theta$, then $\frac{1}{x} = \text{cosec}\,\theta - \cot\theta$.

(3) **Sign of trigonometrical ratios or functions:** Their signs depends on the quadrant in which the terminal side of the angle lies.

(i) **In first quadrant:** $x > 0, y > 0 \Rightarrow \sin\theta = \frac{y}{r} > 0, \cos\theta = \frac{x}{r} > 0, \tan\theta = \frac{y}{x} > 0,\ \text{cosec}\,\theta = \frac{r}{y} > 0$, $\sec\theta = \frac{r}{x} > 0$ and $\cot\theta = \frac{x}{y} > 0$. Thus, in the first quadrant all trigonometric functions are positive.

(ii) **In second quadrant:** $x < 0, y > 0 \Rightarrow \sin\theta = \frac{y}{r} > 0, \cos\theta = \frac{x}{r} < 0, \tan\theta = \frac{y}{x} < 0, \text{cosec}\,\theta = \frac{r}{y} > 0,$ $\sec\theta = \frac{r}{x} < 0$ and $\cot\theta = \frac{x}{y} < 0$. Thus, in the second quadrant sin and cosec function are positive and all others are negative.

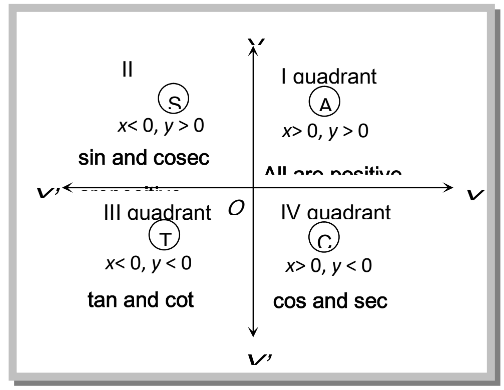
(iii) **In third quadrant:** $x < 0, y < 0 \Rightarrow \sin\theta = \frac{y}{r} < 0, \cos\theta = \frac{x}{r} < 0, \tan\theta = \frac{y}{x} > 0, \text{cosec}\,\theta = \frac{r}{y} < 0$, $\sec\theta = \frac{r}{x} < 0$ and $\cot\theta = \frac{x}{y} > 0$. Thus, in the third quadrant all trigonometric functions are negative except tangent and cotangent.

(iv) **In fourth quadrant:** $x > 0, y < 0 \Rightarrow \sin\theta = \frac{y}{r} < 0,\ \cos\theta = \frac{x}{r} > 0,$

$\tan\theta = \frac{y}{x} < 0,\ \text{cosec}\,\theta = \frac{r}{y} < 0$, $\sec\theta = \frac{r}{x} > 0$ and $\cot\theta = \frac{x}{y} < 0$

Thus, in the fourth quadrant all trigonometric functions are negative except cos and sec.

In brief : A crude aid to memorise the signs of trigonometrical ratio in different quadrant. "**A**dd **S**ugar **T**o **C**offee".



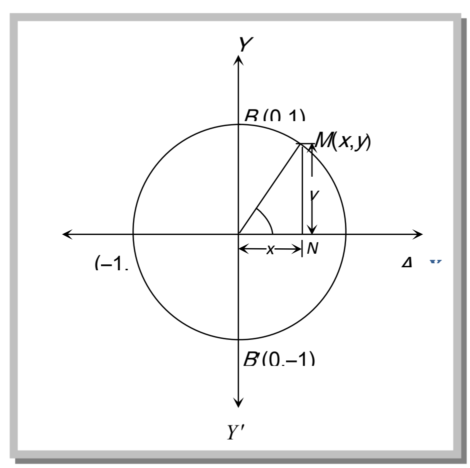
Important Tips

- First determine the sign of the trigonometric function.
- If θ is measured from $X'OX$ i.e., $\{(\pi \pm \theta, 2\pi - \theta)\}$ then retain the original name of the function.
- If θ is measured from $Y'OY$ i.e., $\left\{\frac{\pi}{2} \pm \theta, \frac{3\pi}{2} \pm \theta\right\}$, then change sine to cosine, cosine to sine, tangent to

cotangent, cot to tan, sec to cosec and cosec to sec.

(4) **Variations in values of trigonometric functions in different quadrants:** Let $X'OX$ and YOY' be the coordinate axes. Draw a circle with center at origin O and radius unity.

Let $M(x, y)$ be a point on the circle such that $\angle AOM = \theta$ then $x = \cos\theta$ and $y = \sin\theta$; $-1 \le \cos\theta \le 1$ and $-1 \le \sin\theta \le 1$ for all values of θ.



II-Quadrant (S)	**I-Quadrant (A)**
$\sin\theta \rightarrow$ decreases from 1 to 0	$\sin\theta \rightarrow$ increases from 0 to 1
$\cos\theta \rightarrow$ decreases from 0 to – 1	$\cos\theta \rightarrow$ decreases from

	1 to 0
$\tan\theta \to$ increases from – ∞ to 0	$\tan\theta \to$ increases from 0 to ∞
$\cot\theta \to$ decreases from 0 to – ∞	$\cot\theta \to$ decreases from ∞ to 0
$\sec\theta \to$ increases from – ∞ to – 1	$\sec\theta \to$ increases from 1 to ∞
$\text{cosec}\,\theta \to$ increases from 1 to ∞	$\text{cosec}\,\theta \to$ decreases from ∞ to 1
III-Quadrant (T)	**IV-Quadrant (C)**
$\sin\theta \to$ decreases from 0 to – 1	$\sin\theta \to$ increases from – 1 to 0
$\cos\theta \to$ increases from – 1 to 0	$\cos\theta \to$ increases from 0 to 1
$\tan\theta \to$ increases from 0 to ∞	$\tan\theta \to$ increases from – ∞ to 0
$\cot\theta \to$ decreases from ∞ to 0	$\cot\theta \to$ decreases from 0 to – ∞
$\sec\theta \to$ decreases from – 1 to – ∞	$\sec\theta \to$ decreases from ∞ to 1
$\text{cosec}\,\theta \to$ increases from – ∞ to – 1	$\text{cosec}\,\theta \to$ decreases from – 1 to – ∞

Note: $+\infty$ and – ∞ are two symbols. These are not real number. When we say that tanθincreases from 0 to ∞ for as θvariesfrom 0 to $\frac{\pi}{2}$ it means that $\tan\theta$ increases in the interval $\left(0, \frac{\pi}{2}\right)$ and it attains large positive values as θ tends to $\frac{\pi}{2}$. Similarly for other trigonometric functions.