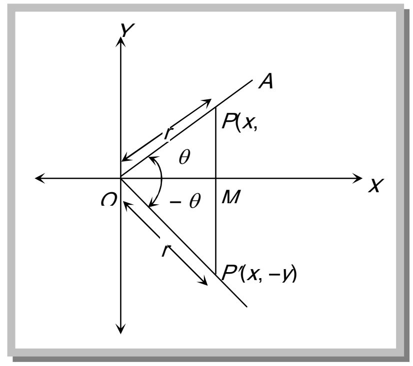
Trigonometrical Ratios of Allied Angles.

Two angles are said to be allied when their sum or difference is either zero or a multiple of 90^o.

(1) **Trigonometric ratios of (–θ):** Let a revolving ray starting from its initial position OX, trace out an angle $\angle XOA = \theta$. Let P(x, y) be a point on OA such that OP = r. Draw PM ⊥ from P on x-axis. Angle $\angle XOA' = -\theta$ in the clockwise sense. Let P' be a point on OA' such that $OP' = OP$. Clearly M and M' coincide and ΔOMP is congruent to $\Delta OMP'$ then P' are (x, – y).



$$\sin(-\theta) = \frac{-y}{r} \Rightarrow \frac{-y}{r} = -\sin\theta; \quad \cos(-\theta) = \frac{x}{r} = \cos\theta; \quad \tan(-\theta) = \frac{-y}{x} = -\tan\theta$$

Taking the reciprocal of these trigonometric ratios;

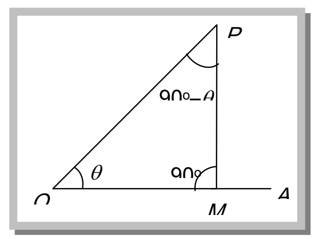
$\text{cosec}(-\theta) = -\text{cosec}\,\theta$, $\quad \sec(-\theta) = \sec\theta$ and $\cot(-\theta) = -\cot\theta$

Note:A function $f(x)$ is said to be an even function if $f(-x) = f(x)$ for all x in its domain.

A function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$ for all x in its domain.

$\sin\theta, \tan\theta, \cot\theta, \text{cosec}\,\theta$ are odd functions and $\cos\theta, \sec\theta$ are even functions.

(2) **Trigonometric function of (90 – θ):** Let the revolving line, starting from OA, trace out any acute angle AOP, equal to θ. From any point P, draw PM ⊥ to OA. Three angles of a triangle are together equal to two right angles, and since OMP is a right angle, the sum of the two angles MOP and OPM is right angle.



$\angle OPM = 90^o - \theta$

[When the angle OPM is consider, the line PM is the 'base' and MO is the 'perpendicular']

$$\sin(90^o - \theta) = \sin MPO = \frac{MO}{PO} = \cos AOP = \cos\theta,$$

$$\cos(90^o - \theta) = \cos MPO = \frac{PM}{PO} = \sin AOP = \sin\theta$$

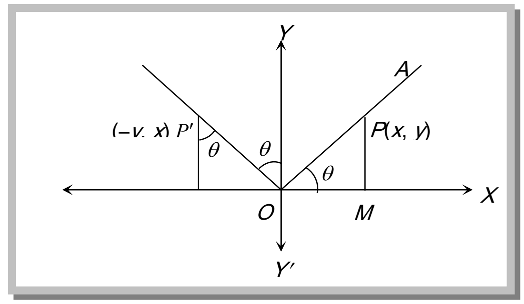
$$\tan(90^o - \theta) = \tan MPO = \frac{MO}{PM} = \cot AOP = \cot\theta,$$

$$\cot(90^o - \theta) = \cot MPO = \frac{PM}{MO} = \tan AOP = \tan\theta$$

$$\text{cosec}(90^o - \theta) = \text{cosec}\ MPO = \frac{PO}{MO} = \sec AOP = \sec\theta,$$

$$\sec(90^o - \theta) = \sec MPO = \frac{PO}{PM} = \text{cosec}\ AOP = \text{cosec}\,\theta$$

(3) **Trigonometric function of (90+θ):** Let a revolving ray OA starting from its initial position OX, trace out an angle $\angle XOA = \theta$ and let another revolving ray OA' starting from the same initial position OX, first trace out an angle θ. So as to coincide with OA and then it revolves through an angle of 90^o in anticlockwise direction to form an angle $\angle XOA' = 90^o + \theta$.



Let P and P' be points on OA and OA' respectively such that $OP = OP' = r$.

Draw perpendicular PM and PM' from P and P' respectively on OX. Let the coordinates of P be (x, y). Then $OM = x$ and $PM = y$ clearly, $OM' = PM = y$ and $P'M' = OM = x$.

So the coordinates of P' are –y, x

$$\sin(90+\theta) = \frac{M'P'}{OP'} = \frac{x}{r} = \cos\theta,\ \cos(90+\theta) = \frac{OM'}{OP'} = \frac{-y}{r} = -\sin\theta$$

$$\tan(90+\theta) = \frac{M'P'}{OM'} = \frac{x}{-y} = \frac{-x}{y} = -\cot\theta,\ \cot(90+\theta) = -\tan\theta,\ \sec(90+\theta) = -\text{cosec}\,\theta,$$

$$\text{cosec}(90+\theta) = \sec\theta$$

Allied angles → / **Trigo. Ratio ↓**	$(-\theta)$	**(90 – θ)** or $\left(\frac{\pi}{2}-\theta\right)$	$(90+\theta)$ or $\left(\frac{\pi}{2}+\theta\right)$	$(180-\theta)$ or $(\pi-\theta)$	$(180+\theta)$ or $(\pi+\theta)$	$(270-\theta)$ or $\left(\frac{3\pi}{2}-\theta\right)$	$(270+\theta)$ or $\left(\frac{3\pi}{2}+\theta\right)$	$(360-\theta)$ or $(2\pi-\theta)$
sinθ	– sinθ	cosθ	$\cos\theta$	sinθ	– sinθ	– cosθ	– cos θ	– sinθ
cosθ	cosθ	sinθ	– sinθ	– cosθ	– cosθ	– sinθ	sinθ	cosθ
tan θ	– tanθ	cotθ	– cotθ	– tanθ	tanθ	cotθ	– cotθ	– tanθ

Important Tips

- $\sin n\pi = 0,\ \cos n\pi = (-1)^n$
- $\sin(n\pi+\theta) = (-1)^n \sin\theta$, $\cos(n\pi+\theta) = (-1)^n \cos\theta$

☞ $\sin\left(\frac{n\pi}{2}+\theta\right)=(-1)^{\frac{n-1}{2}}\cos\theta$, if n is odd

$=(-1)^{n/2}\sin\theta$, if n is even

☞ $\cos\left(\frac{n\pi}{2}+\theta\right)=(-1)^{\frac{n+1}{2}}\sin\theta,\ \text{if } n \text{ is odd}$

$=(-1)^{n/2}\cos\theta,\quad \text{if } n \text{ is even}$