Periodic Functions.

A function f(x) is called periodic function if there exists a least positive real number T such that $f(x+T)=f(x)$. T is called the period (or fundamental period) of function $f(x)$. Obviously, if T is the period of $f(x)$, then $f(x)=f(x+T)=f(x+2T)=f(x+3T)=$ ....

(i) If $f_1(x)$ and $f_2(x)$ are two periodic functions of x having the same period T, then the function $af_1(x)+bf_2(x)$ where a and b are any numbers, is also a periodic function having the same period T.

(ii) If T is the period of the periodic function $f(x)$, then the function $f(ax+b)$, where $a(>0)$ and b are any numbers is also a periodic function with period equal to T/a.

(iii) If T_1 and T_2 are the periods of periodic functions $f_1(x)$ and $f_2(x)$ respectively, then the function $af_1(x)+bf_2(x)$, where a and b are any numbers is also periodic and its period is T which is the L.C.M. of T_1 and T_2 i.e. T is the least positive number which is divisible by T_1 and T_2.

All trigonometric functions are periodic. The period of trigonometric function $\sin x, \cos x, \sec x$ and $co\sec x$ is 2π because $\sin(x+2\pi)=\sin x, \cos(x+2\pi)=\cos x$ etc.

The period of $\tan x$ and $\cot x$ is π because $\tan(x+\pi)=\tan x$ and $\cot(x+\pi)=\cot x$

The period of the function which are of the type: $\sin ax, \cos(ax+b); b\cos ax$ is $\frac{2\pi}{a}$

The period of $\tan ax$ and $\cot ax$ is $\frac{\pi}{|a|}$. Here |a| is taken so as the value of the period is positive real number.

Some functions with their periods

Function	Period
$\sin(ax+b), \cos(ax+b), \sec(ax+b), \text{cosec}(ax+b)$	$2\pi / a$
$\tan(ax+b), \cot(ax+b)$	π / a
$\lvert \sin(ax+b)\rvert, \lvert \cos(ax+b)\rvert, \lvert \sec(ax+b)\rvert, \lvert \text{cosec}(ax+b)\rvert$	π / a
$\lvert \tan(ax+b)\rvert, \lvert \cot(ax+b)\rvert$	π / a