General Solution of Standard Trigonometrical Equations.

(1) **General solution of the equation sin θ = sinα:** If $\sin\theta = \sin\alpha$ or $\sin\theta - \sin\alpha = 0$

or, $\frac{\theta-\alpha}{2} = m\pi; m \in I$ or $\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2}; m \in I$

$\Rightarrow \theta = 2m\pi + \alpha; m \in I$ or $\theta = (2m+1)\pi - \alpha; m \in I$

$\Rightarrow \theta$ = (any even multiple of π) + α or θ = (any odd multiple of π) – α

$$\boxed{\theta = n\pi + (-1)^n \alpha \; ; n \in I}$$

Note: The equation $\operatorname{cosec}\theta = \operatorname{cosec}\alpha$ is equivalent to $\sin\theta = \sin\alpha$. So these two equation having the same general solution.

(2) **General solution of the equation cosθ = cosα :** If $\cos\theta = \cos\alpha \Rightarrow \cos\theta - \cos\alpha = 0 \Rightarrow$ $-2\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\theta-\alpha}{2}\right) = 0 \Rightarrow \sin\left(\frac{\theta+\alpha}{2}\right) = 0$ or $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$, $\Rightarrow \frac{\theta+\alpha}{2} = n\pi; n \in I$ or $\frac{\theta-\alpha}{2} = n\pi; n \in I$

$\Rightarrow \theta = 2n\pi - \alpha; n \in I$ or $\theta = 2n\pi + \alpha; n \in I$. for the general solution of $\cos\theta = \cos\alpha$, combine these two result which gives $\boxed{\theta = 2n\pi \pm \alpha; \; n \in I}$

Note: The equation $\sec\theta = \sec\alpha$ is equivalent to $\cos\theta = \cos\alpha$, so the general solution of these two equations are same.

(3) **General solution of the equation tan θ = tan α :** If $\tan\theta = \tan\alpha \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \sin\theta\cos\alpha - \cos\theta\sin\alpha = 0 \Rightarrow \sin(\theta-\alpha) = 0 \Rightarrow \theta - \alpha = n\pi; n \in I$ $\boxed{\theta = n\pi + \alpha; n \in I}$

Note: The equation $\cot\theta = \cot\alpha$ is equivalent to $\tan\theta = \tan\alpha$ so these two equations having the same general solution.