General Solution of the form a cosθ + bsinθ = c.

In $a\cos\theta + b\sin\theta = c,$ put $a = r\cos\alpha$ and $b = r\sin\alpha$ where $r = \sqrt{a^2 + b^2}$ and $|c| \le \sqrt{a^2 + b^2}$

Then, $r(\cos\alpha\cos\theta + \sin\alpha\sin\theta) = c \Rightarrow \cos(\theta - \alpha) = \dfrac{c}{\sqrt{a^2 + b^2}} = \cos\beta$ (say) ....(i)

$\Rightarrow \theta - \alpha = 2n\pi \pm \beta \Rightarrow \theta = 2n\pi \pm \beta + \alpha,$ where $\tan\alpha = \dfrac{b}{a},$ is the general solution

Alternatively, putting $a = r\sin\alpha$ and $b = r\cos\alpha$ where $r = \sqrt{a^2 + b^2}$

$\Rightarrow \sin(\theta + \alpha) = \dfrac{c}{\sqrt{a^2 + b^2}} = \sin\gamma$ (say)

$\Rightarrow \theta + \alpha = n\pi + (-1)^n\gamma \Rightarrow \theta = n\pi + (-1)^n\gamma - \alpha,$ where $\tan\alpha = \dfrac{a}{b}$, is the general solution.

Note: $(-\sqrt{a^2 + b^2}) \le a\cos\theta + b\sin\theta \le (\sqrt{a^2 + b^2})$

The general solution of $a\cos x + b\sin x = c$ is $x = 2n\pi + \tan^{-1}\left(\dfrac{b}{a}\right) \pm \cos^{-1}\left(\dfrac{c}{\sqrt{a^2 + b^2}}\right).$