Some Particular Equations.

(1) **Equation of the form** $\boldsymbol{a_0 \sin^n x + a_1 \sin^{n-1} x \cos x + a_2 \sin^{n-2} x \cos^2 x + \dots + a_n \cos^n x = 0}$ **:** Here $a_0, a_1 \dots, a_n$ are real numbers and the sum of the exponents in $\sin x$ and $\cos x$ in each term is equal to n, are said to be homogeneous with respect to sinx and cosx. For $\cos x \neq 0,$ above equation can be written as, $a_0 \tan^n x + a_1 \tan^{n-1} x + \dots + a_n = 0.$

(2) **A trigonometric equation of the form** $\boldsymbol{R(\sin kx, \cos nx, \tan mx, \cot lx) = 0}$ **:** Here R is a rational function of the indicated arguments and (k, l, m, n are natural numbers) can be reduced to a rational equation with respect to the arguments $\sin x, \cos x, \tan x,$ and $\cot x$ by means of the formulae for trigonometric functions of the sum of angles (in particular, the formulas for double and triple angles) and then reduce equation of the given form to a rational equation with respect to the unknown, $t = \tan \frac{x}{2}$ by means of the formulas,

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}},$$

(3) **Equation of the form** $\boldsymbol{R(\sin x + \cos x, \sin x . \cos x) = 0}$ **:** where R is rational function of the arguments in brackets, Put $\sin x + \cos x = t$ ....(i) and use the following identity:

$$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x \Rightarrow \sin x \cos x = \frac{t^2 - 1}{2} \quad \text{.......(ii)}$$

Taking (i) and (ii) into account, we can reduce given equation into; $R\left(t, \frac{t^2 - 1}{2}\right) = 0$.

Similarly, by the substitution $(\sin x - \cos x) = t,$ we can reduce the equation of the form;

$R(\sin x - \cos x, \sin x \cos x) = 0$ to an equation; $R\left(t, \frac{1 - t^2}{2}\right) = 0.$