Important Points to be taken in Case of While Solving Trigonometrical Equations.

(1)Check the validity of the given equation, e.g., $2\sin\theta - \cos\theta = 4$ can never be true for any θ as the value $(2\sin\theta - \cos\theta)$ can never exceeds $\sqrt{2^2 + (-1)^2} = \sqrt{5}$. So there is no solution to this equation.

(2) Equation involving $\sec\theta$ or $\tan\theta$ can never have a solution of the form. $(2n+1)\frac{\pi}{2}$

Similarly, equations involving $\operatorname{cosec}\theta$ or $\cot\theta$ can never have a solution of the form $\theta = n\pi$. The corresponding functions are undefined at these values of θ.

(3) If while solving an equation we have to square it, then the roots found after squaring must be checked whether they satisfy the original equation or not, e.g., Let $x = 3$. Squaring, we get $x^2 = 9 \therefore x = 3$ and -3 but $x = -3$ does not satisfy the original equation $x = 3$. e.g.,

$\sin x + \cos x = 1$

Square both sides, we get $1 + \sin 2x = 1$ $\quad \therefore \sin 2x = 0$

$\therefore 2x = n\pi$ or $x = \frac{n\pi}{2}, \; n \in I$

$\therefore$ Roots are $\ldots\ldots, \frac{-3\pi}{2}, \frac{-2\pi}{2}, \frac{-\pi}{2}, 0, \frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}, \ldots\ldots$

We find that 0 and $\pi/2$ are roots but π and $3\pi/2$ do not satisfy the given equation as it leads to $-1 = 1$

Similarly 0 and $\frac{-3\pi}{2}$ are roots but $-\frac{\pi}{2}$ and $-\pi$ are not roots as it will lead to $-1 = 1$.

As stated above, because of squaring we are solving the equations $\sin x + \cos x = 1$ and $\sin x + \cos x = -1$ both. The rejected roots are for $\sin x + \cos x = -1$.

(4) Do not cancel common factors involving the unknown angle on L.H.S. and R.H.S. because it may delete some solutions. e.g., In the equation $\sin\theta(2\cos\theta - 1) = \sin\theta\cos^2\theta$ if we cancel $\sin\theta$ on both sides we get $\cos^2\theta - 2\cos\theta + 1 = 0 \Rightarrow (\cos\theta - 1)^2 = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 2n\pi$. But $\theta = n\pi$ also satisfies the equation because it makes $\sin\theta = 0$. So, the complete solution is $\theta = n\pi, n \in Z$.

(5) Any value of x which makes both R.H.S. and L.H.S. equal will be a root but the value of x for which $\infty = \infty$ will not be a solution as it is an indeterminate form.
Hence, $\cos x \neq 0$ for those equations which involve $\tan x$ and $\sec x$ whereas $\sin x \neq 0$ for those which involve $\cot x$ and $\text{cosec } x$.
Also exponential function is always +ve and $\log_a x$ is defined if $x > 0$, $x \neq 0$ and $a > 0, a \neq 1$
$\sqrt{f(x)} = +ve$ always and not $\pm$ *i.e.* $\sqrt{(\tan^2 x)} = \tan x$ and not $\pm \tan x$.

(6) Denominator terms of the equation if present should never become zero at any stage while solving for any value of θ contained in the answer.

(7) Sometimes the equation has some limitations also e.g., $\cot^2 \theta + \text{cosec}^2\theta = 1$ can be true only if $\cot^2 \theta = 0$ and $\text{cosec}^2\theta = 1$ simultaneously as $\text{cosec}^2\theta \geq 1$. Hence the solution is $\theta = (2n+1)\pi / 2$.

(8) If $xy = xz$ then $x(y-z) = 0 \Rightarrow$ either $x = 0$ or $y = z$ or both. But $\frac{y}{x} = \frac{z}{x} \Rightarrow y = z$ only and not $x = 0$, as it will make $\infty = \infty$. Similarly if $ay = az$, then it will also imply $y = z$ only as $a \neq 0$ being a constant.
Similarly $x + y = x + z \Rightarrow y = z$ and $x - y = x - z \Rightarrow y = z$. Here we do not take $x = 0$ as in the above because x is an additive factor and not multiplicative factor.
When $\cos\theta = 0$, then $\sin\theta = 1$ or -1. We have to verify which value of $\sin\theta$ is to be chosen which satisfies the equation. $\cos\theta = 0 \Rightarrow \theta = \left(n + \frac{1}{2}\right)\pi$.

(9) Student are advised to check whether all the roots obtained by them, satisfy the equation and lie in the domain of the variable of the given equation.