Formulae for Sum and Difference of Inverse Trigonometric Function.

(1) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; If $x > 0, y > 0$ and $xy < 1$

(2) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; If $x > 0, y > 0$ and $xy > 1$

(3) $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; If $x < 0, y < 0$ and $xy > 1$

(4) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; If $xy > -1$

(5) $\tan^{-1} - \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; If $x > 0, y < 0$ and $xy < -1$

(6) $\tan^{-1} x - \tan^{-1} y = -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; If $x < 0, y > 0$ and $xy < -1$

(7) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(8) $\tan^{-1} x_1 + \tan^{-1} x_2 + ... + \tan^{-1} x_n = \tan^{-1}\left[\frac{S_1 - S_3 + S_5 - ....}{1 - S_2 + S_4 - S_6 + ...}\right]$,

Where S_k denotes the sum of the products of $x_1, x_2, ..., x_n$ taken k at a time.

(9) $\cot^{-1} x + \cot^{-1} y = \cot^{-1} \frac{xy-1}{y+x}$

(10) $\cot^{-1} x - \cot^{-1} y = \cot^{-1} \frac{xy+1}{y-x}$

(11) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$;

If $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$ or if $xy < 0$ and $x^2 + y^2 > 1$

(12) $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$, If $0 < x, y \le 1$ and $x^2 + y^2 > 1$

(13) $\sin^{-1} x + \sin^{-1} y = -\pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$, If $-1 \le x; y < 0$ and $x^2 + y^2 > 1$

(14) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$, If $-1 \le x; y \le 1$ and $x^2 + y^2 \le 1$ if or $xy > 0$ and $x^2 + y^2 > 1$.

(15) $\sin^{-1} x - \sin^{-1} y = \pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$, If $0 < x \le 1, -1 \le y < 0$ and $x^2 + y^2 > 1$.

(16) $\sin^{-1} x - \sin^{-1} y = -\pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$, If $-1 \le x < 0, 0 < y \le 1$ and $x^2 + y^2 > 1$.

(17) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\{xy - \sqrt{1-x^2}.\sqrt{1-y^2}\}$, If $-1 \le x, y \le 1$ and $x + y \ge 0$.

(18) $\cos^{-1} x + \cos^{-1} y = 2\pi - \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$, If $-1 \le x, y \le 1$ and $x + y \le 0$

(19) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}$, If $-1 \le x, y \le 1$, and $x \le y$.

(20) $\cos^{-1} x - \cos^{-1} y = -\cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$, If $-1 \le y \le 0,\ 0 < x \le 1$ and $x \ge y$.

Important Tips

- If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then $xy + yz + zx = 1$.
- If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then $x + y + z = xyz$.
- If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$, then $x^2 + y^2 + z^2 + 2xyz = 1$.
- If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.
- If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx = 3$.
- If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then $x^2 + y^2 + z^2 + 2xyz = 1$.
- If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then $xy + yz + zx = 3$.

- If $\sin^{-1} x + \sin^{-1} y = \theta$, then $\cos^{-1} x + \cos^{-1} y = \pi - \theta$.
- If $\cos^{-1} x + \cos^{-1} y = \theta$, then $\sin^{-1} x + \sin^{-1} y = \pi - \theta$.
- If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{2}$, then $xy = 1$.
- If $\cot^{-1} x + \cot^{-1} y = \frac{\pi}{2}$, then $xy = 1$.
- If $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \theta$, then $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos\theta + \frac{y^2}{b^2} = \sin^2 \theta$.