

Definition.

We know that parametric co-ordinates of any point on the unit circle $x^2 + y^2 = 1$ is $(\cos \theta, \sin \theta)$; so that these functions are called circular functions and co-ordinates of any point on unit

hyperbola $x^2 - y^2 = 1$ is $\left(\frac{e^\theta + e^{-\theta}}{2}, \frac{e^\theta - e^{-\theta}}{2}\right)$ i.e., $(\cosh \theta, \sinh \theta)$. It means that the relation

which exists amongst $\cos \theta, \sin \theta$ and unit circle, that relation also exist amongst $\cosh \theta, \sinh \theta$ and unit hyperbola. Because of this reason these functions are called as Hyperbolic functions.

For any (real or complex) variable quantity x ,

$$(1) \sinh x = \frac{e^x - e^{-x}}{2} \text{ [Read as 'hyperbolic sine x']}$$

$$(2) \cosh x = \frac{e^x + e^{-x}}{2} \text{ [Read as 'hyperbolic cosine x']}$$

$$(3) \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$(4) \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$(5) \operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$(6) \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

Note: $\sinh 0 = 0, \cosh 0 = 1, \tanh 0 = 0$