

Formulae for Hyperbolic Functions.

The following formulae can easily be established directly from above definitions

(1) Reciprocal formulae

$$(i) \operatorname{cosech} x = \frac{1}{\sinh x}$$

$$(ii) \operatorname{sech} x = \frac{1}{\cosh x}$$

$$(iii) \operatorname{coth} x = \frac{1}{\tanh x}$$

$$(iv) \tanh x = \frac{\sinh x}{\cosh x}$$

$$(v) \operatorname{coth} x = \frac{\cosh x}{\sinh x}$$

(2) Square formulae

$$(i) \cosh^2 x - \sinh^2 x = 1$$

$$(ii) \operatorname{sech}^2 x + \tanh^2 x = 1$$

$$(iii) \operatorname{coth}^2 x - \operatorname{cosech}^2 x = 1$$

$$(iv) \cosh^2 x + \sinh^2 x = \cosh 2x$$

(3) Expansion or Sum and difference formulae

$$(i) \sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \quad (ii) \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$(iii) \tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

(4) Formulae to transform the product into sum or difference

$$(i) \sinh x + \sinh y = 2 \sinh \frac{x+y}{2} \cosh \frac{x-y}{2}$$

$$(ii) \sinh x - \sinh y = 2 \cosh \frac{x+y}{2} \sinh \frac{x-y}{2}$$

$$(iii) \cosh x + \cosh y = 2 \cosh \frac{x+y}{2} \cosh \frac{x-y}{2}$$

$$(iv) \cosh x - \cosh y = 2 \sinh \frac{x+y}{2} \sinh \frac{x-y}{2}$$

$$(v) 2 \sinh x \cosh y = \sinh (x + y) + \sinh (x - y)$$

$$(vi) 2 \cosh x \sinh y = \sinh (x + y) - \sinh (x - y)$$

$$(vii) 2 \cosh x \cosh y = \cosh (x + y) + \cosh (x - y)$$

$$(viii) 2 \sinh x \sinh y = \cosh (x + y) - \cosh (x - y)$$

$$(ix) \cosh x + \sinh x = e^x$$

$$(x) \cosh x - \sinh x = e^{-x}$$

$$(xi) (\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$

(5) Trigonometric ratio of multiple of an angle

$$(i) \sinh 2x = 2 \sinh x \cosh x = \frac{2 \tanh x}{1 - \tanh^2 x}$$

$$(ii) \cosh 2x = \cosh^2 x + \sinh^2 x = 2 \cosh^2 x - 1 = 1 + 2 \sinh^2 x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x}$$

$$(iii) 2 \cosh^2 x = \cosh 2x + 1$$

$$(iv) 2 \sinh^2 x = \cosh 2x - 1$$

$$(v) \tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

$$(vi) \sinh 3x = 3 \sinh x + 4 \sinh^3 x$$

$$(vii) \cosh 3x = 4 \cosh^3 x - 3 \cosh x$$

$$(viii) \tanh 3x = \frac{3 \tanh x + \tanh^3 x}{1 + 3 \tanh^2 x}$$

(6)

$$(i) \cosh x + \sinh x = e^x$$

$$(ii) \cosh x - \sinh x = e^{-x}$$

$$(iii) (\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$