Conditional Probability.

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and P(B) ≠ 0, is called the conditional probability and it is denoted by P(A/B).

Thus, P(A/B) = Probability of occurrence of A, given that B has already happened.

$$= \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}.$$

Similarly, P(B/A) = Probability of occurrence of B, given that A has already happened.

$$= \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)}.$$

Note: Sometimes, P(A/B) is also used to denote the probability of occurrence of A when B occurs. Similarly, P(B/A) is used to denote the probability of occurrence of B when A occurs.

(1) **Multiplication theorems on probability**

(i) If A and B are two events associated with a random experiment, then $P(A \cap B) = P(A).P(B/A)$, if P(A) ≠ 0 or $P(A \cap B) = P(B).P(A/B)$, if P(B) ≠ 0.

(ii) **Extension of multiplication theorem:** If $A_1, A_2, ..., A_n$ are n events related to a random experiment, then

$$P(A_1 \cap A_2 \cap A_3 \cap .... \cap A_n) = P(A_1)P(A_2/A_1)P(A_3/A_1 \cap A_2)....P(A_n/A_1 \cap A_2 \cap ... \cap A_{n-1}),$$

Where $P(A_i/A_1 \cap A_2 \cap ... \cap A_{i-1})$ represents the conditional probability of the event A_i, given that the events $A_1, A_2, ..., A_{i-1}$ have already happened.

(iii) **Multiplication theorems for independent events:** If A and B are independent events associated with a random experiment, then $P(A \cap B) = P(A).P(B)$ i.e., the probability of simultaneous occurrence of two independent events is equal to the product of their probabilities.

By multiplication theorem, we have $P(A \cap B) = P(A).P(B/A)$.

Since A and B are independent events, therefore $P(B/A) = P(B)$. Hence, $P(A \cap B) = P(A).P(B)$.

(iv) **Extension of multiplication theorem for independent events:**If $A_1, A_2, ..., A_n$ are independent events associated with a random experiment, then
$P(A_1 \cap A_2 \cap A_3 \cap ... \cap A_n) = P(A_1)P(A_2)...P(A_n)$.
By multiplication theorem, we have
$P(A_1 \cap A_2 \cap A_3 \cap ... \cap A_n) = P(A_1)P(A_2 / A_1)P(A_3 / A_1 \cap A_2)...P(A_n / A_1 \cap A_2 \cap ... \cap A_{n-1})$
Since $A_1, A_2, ..., A_{n-1}, A_n$ are independent events, therefore
$P(A_2 / A_1) = P(A_2), P(A_3 / A_1 \cap A_2) = P(A_3), ..., P(A_n / A_1 \cap A_2 \cap ... \cap A_{n-1}) = P(A_n)$
Hence, $P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1)P(A_2)....P(A_n)$.

(2) **Probability of at least one of the n independent events:**If $p_1, p_2, p_3, ..., p_n$ be the probabilities of happening of n independent events $A_1, A_2, A_3, ..., A_n$ respectively, then

(i) Probability of happening none of them
$= P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 ...... \cap \overline{A}_n) = P(\overline{A}_1).P(\overline{A}_2).P(\overline{A}_3).....P(\overline{A}_n) = (1 - p_1)(1 - p_2)(1 - p_3)....(1 - p_n)$.

(ii) Probability of happening at least one of them
$= P(A_1 \cup A_2 \cup A_3 .... \cup A_n) = 1 - P(\overline{A}_1)P(\overline{A}_2)P(\overline{A}_3)....P(\overline{A}_n) = 1 - (1 - p_1)(1 - p_2)(1 - p_3)...(1 - p_n)$.

(iii) Probability of happening of first event and not happening of the remaining
$= P(A_1)P(\overline{A}_2)P(\overline{A}_3).....P(\overline{A}_n) = p_1(1 - p_2)(1 - p_3).......(1 - p_n)$