Angle between Two lines of Regression.

Equation of the two lines of regression are $y-\bar{y}=b_{yx}(x-\bar{x})$ and $x-\bar{x}=b_{xy}(y-\bar{y})$

We have, $m_1=$ slope of the line of regression of y on x = $b_{yx}=r.\frac{\sigma_y}{\sigma_x}$

$m_2=$ Slope of line of regression of x on y = $\frac{1}{b_{xy}}=\frac{\sigma_y}{r.\sigma_x}$

$$\therefore \tan\theta=\pm\frac{m_2-m_1}{1+m_1m_2}=\pm\frac{\frac{\sigma_y}{r\sigma_x}-\frac{r\sigma_y}{\sigma_x}}{1+\frac{r\sigma_y}{\sigma_x}.\frac{\sigma_y}{r\sigma_x}}=\pm\frac{(\sigma_y-r^2\sigma_y)\sigma_x}{r\sigma_x^2+r\sigma_y^2}=\pm\frac{(1-r^2)\sigma_x\sigma_y}{r(\sigma_x^2+\sigma_y^2)}.$$

Here the positive sign gives the acute angle θ, because $r^2\le 1$ and σ_x,σ_y are positive.

$$\therefore \tan\theta=\frac{1-r^2}{r}.\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2} \qquad \text{.....(i)}$$

Note: If $r=0$, from (i) we conclude $\tan\theta=\infty$ or $\theta=\pi/2$ i.e., two regression lines are at right angels.

If $r=\pm 1$, $\tan\theta=0$ i.e., $\theta=0$, since θ is acute i.e., two regression lines coincide.