2. Continuity of a Function at a Point.

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain iff $\lim_{x \to a} f(x) = f(a)$. *i.e.* a function $f(x)$ is continuous at $x = a$ if and only if it satisfies the following three conditions:

(1) $f(a)$ exists. ('*a*' lies in the domain of *f*)

(2) $\lim_{x \to a} f(x)$ exist *i.e.* $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$ or R.H.L. = L.H.L.

(3) $\lim_{x \to a} f(x) = f(a)$ (limit equals the value of function).

Cauchy's definition of continuity: A function f is said to be continuous at a point *a* of its domain *D* if for every $\varepsilon > 0$ there exists $\delta > 0$ (dependent on ε) such that
$|x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon.$

Comparing this definition with the definition of limit we find that $f(x)$ is continuous at $x = a$ if $\lim_{x \to a} f(x)$ exists and is equal to $f(a)$ *i.e.*, if $\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a+} f(x)$.

Heine's definition of continuity: A function f is said to be continuous at a point *a* of its domain *D*, converging to *a*, the sequence $< a_n >$ of the points in *D* converging to *a*, the sequence $< f(a_n) >$ converges to $f(a)$ *i.e.* $\lim a_n = a \Rightarrow \lim f(a_n) = f(a)$. This definition is mainly used to prove the discontinuity to a function.

Note: Continuity of a function at a point, we find its limit and value at that point, if these two exist and are equal, then function is continuous at that point.

Formal definition of continuity: The function $f(x)$ is said to be continuous at $x = a$, in its domain if for any arbitrary chosen positive number $\in > 0$, we can find a corresponding number δ depending on $\in$ such that $|f(x) - f(a)| < \in \ \forall x$ for which $0 \nless |x - a| < \delta$.